UNIVERSITY OF SOUTH FLORIDA

Relationships Between Important Distributions in Communication Channels

Color Codes: Fundamental Dist. | Amplitude Dist. | Power Dist.

Gauss Distribution:

$$f(x)=\frac{1}{\sqrt{2\pi\sigma_p^2}}e^{-\frac{(x-\mu_p)^2}{2\sigma_p^2}}$$

$$\mu=\mu_p$$
$$\sigma^2=\sigma_p^2$$

Gauss → Lognormal: $y=e^x$

Gauss → Rayleigh: $y=\sqrt{x_1^2+x_2^2}$ Jacobian

Lognormal Distribution:

$$f(x)=\frac{1}{x\sigma_p\sqrt{2\pi}}e^{-\frac{(\ln(x)-\mu_p)^2}{2\sigma_p^2}}$$

$$\mu=e^{\mu_p+\frac{\sigma_p^2}{2}}$$

$$\sigma^2=(e^{\sigma_p^2}-1)e^{2\mu_p+\sigma_p^2}$$

Lognormal/Gauss → Shadowing Distribution (Power): $y=x^{\frac{ln10}{10}}=x^{\xi}$ (when x is lognormal) or $y=10^{x/10}=e^{\frac{xln10}{10}}=e^{\xi x}$(when x is Gaussian)

Lognormal/Gauss → Shadowing Distribution (Amplitude): $y=x^{\frac{1}{2}\frac{ln10}{10}}=x^{\frac{\xi}{2}}$ (when x is lognormal) or $y=\sqrt{10^{x/10}}=e^{\frac{1}{2}\frac{xln10}{10}}=e^{\frac{\xi}{2}x}$(when x is Gaussian)

Rayleigh Distribution:

$$f(x)=\frac{x}{\sigma_p^2}e^{-\frac{x^2}{2\sigma^2}}$$

$$\mu=\sigma_p\sqrt{\frac{\pi}{2}}$$

$$\sigma^2=\left(\frac{4-\pi}{2}\right)\sigma_p^2$$

$$\Omega_p=\mu^2+\sigma^2=2\sigma_p$$

Rician → Rayleigh: $K=0$

Nakagami-m → Rician: $m\cong\frac{(K+1)^2}{2K+1}$ No Bessel in Nakagami-m

Nakagami-m → Rayleigh: $m=1$

Rayleigh → Exponential: $\lambda=\Omega_p=2\sigma_p$

Rician Distribution:

$$f(x)=\frac{x}{b_0}e^{-\frac{x^2+s^2}{2b_0}}I_0\left(\frac{xs}{b_0}\right)$$

Specular Power: $s^2=m_I^2(t)+m_Q^2(t)$

Diffuse Power: $2b_0$

$$\Omega_p=s^2+2b_0, K=\frac{s^2}{2b_0}$$

$$f(x)=\frac{2x(K+1)}{\Omega_p}e^{-K-\frac{(K+1)x^2}{\Omega_p}}I_0\left(2x\sqrt{\frac{K(K+1)}{\Omega_p}}\right)$$

Nakagami-m Distribution:

$$f(x)=\frac{2m^m x^{2m-1}}{\Gamma(m)\Omega_p^m}e^{-\frac{mx^2}{\Omega_p}}$$

$$m\geq\frac{1}{2}$$

$$\mu=\frac{\Gamma\left(m+\frac{1}{2}\right)}{\Gamma(m)}\left(\frac{\Omega_p}{m}\right)^{\frac{1}{2}}$$

$$\sigma^2=\Omega_p-\frac{\Omega_p}{m}\left(\frac{\Gamma\left(m+\frac{1}{2}\right)}{\Gamma(m)}\right)^2$$

Nakagami-m → Gamma: $\theta=\frac{\Omega_p}{m}$

Shadowing Distribution (Amplitude)

$$f(x)=\frac{2}{x\sigma_p\xi\sqrt{2\pi}}e^{-\frac{(\log_{10}x^2-\mu_p)^2}{2\sigma_p^2}}$$

Exponential Distribution:

$$f(x)=\lambda e^{-\lambda x}$$

$$\mu=\frac{1}{\lambda}$$

$$\sigma^2=\frac{1}{\lambda^2}$$

Gamma → Exponential: $k=1,\theta=\frac{1}{\lambda}$

Gamma Distribution:

$$f(x)=\frac{x^{k-1}e^{-\frac{x}{\theta}}}{\theta^k\Gamma(k)}$$

$$\mu=k\theta$$
$$\sigma^2=k\theta^2$$

Shadowing Distribution (Power)

$$f(x)=\frac{1}{x\sigma_p\xi\sqrt{2\pi}}e^{-\frac{(\log_{10}x-\mu_p)^2}{2\sigma_p^2}}$$

Composite Distributions:

Suzuki Distribution (Rayleigh+Shadowing):

$$f(x)=\int_0^\infty f_{x|\Omega}\,f_\Omega(\Omega)d\Omega$$

Rayleigh Fading ➔ $\mu=\sigma_p\sqrt{\frac{\pi}{2}}=\Omega$ ➔ $\sigma_p=\Omega\sqrt{\frac{2}{\pi}}$

$$f(x)=\int_0^\infty \frac{x}{\sigma_p^2}e^{-\frac{x^2}{2\sigma^2}}\bigg|_{\sigma_p=\Omega\sqrt{\frac{2}{\pi}}}\frac{2}{\Omega\sigma_p\xi\sqrt{2\pi}}e^{-\frac{(\log_{10}\Omega^2-\mu_p)^2}{2\sigma_p^2}}d\Omega$$

$$f(x)=\int_0^\infty \frac{\pi x}{2\Omega^2}e^{\frac{-\pi x^2}{4\Omega^2}}\frac{2}{\Omega\sigma_p\xi\sqrt{2\pi}}e^{-\frac{(\log_{10}\Omega^2-\mu_p)^2}{2\sigma_p^2}}d\Omega$$

Gamma-Lognormal Distribution: (Nakagami-m + Shadowing):

$$f(x)=\int_0^\infty\frac{\left(\frac{m}{\Omega_p}\right)^m x^{m-1}}{\Gamma(m)}e^{-\frac{mx}{\Omega_p}}\frac{1}{\Omega_p\sigma_p\xi\sqrt{2\pi}}e^{-\frac{(\log_{10}\Omega_p-\mu_p)^2}{2\sigma_p^2}}d\Omega_p$$

$$\cong\frac{1}{x\sigma\sqrt{2\pi}}e^{-\frac{(\ln(x)-\mu)^2}{2\sigma_p^2}}$$

$\mu=\xi^{-1}[\psi(m)-\ln(m)]+\mu_p,\ \sigma^2=\xi^{-2}\zeta(2,m)+\sigma_p^2$

$\psi(\cdot)$: Euler psi function, $\zeta(\cdot)$: Riemann's zeta function

If m=1, valid for $\sigma_p\geq 6\,dB$

If m=2, valid for all σ_p

Alphan Sahin | Wireless Communications and Signal Processing Group | E-mail:alphan@mail.usf.edu