MIMO-OFDM Channel Estimation for Correlated Fading Channels

Mehmet Kemal Özdemir
Syracuse University,
Syracuse, NY, USA
Email: mkozdemi@syr.edu

Hüseyin Arslan
University of South Florida,
Tampa, FL, USA
Email: arslan@eng.usf.edu

Ercüment Arvas
Syracuse University,
Syracuse, NY, USA
Email: earvas@syr.edu

Abstract

This paper presents MIMO-OFDM channel estimation for spatially correlated channels using frequency domain estimation techniques. First, the exploitation of spatial correlation on further improving the channel estimates is investigated. It is observed that spatial filtering provides additional gain when the spatial correlation and the number of antennas are high. For example, in a system with 16 *transmit antennas, the use of additional filtering in spatial domain improves the performance of mean square error (MSE) by* 2 *dB and that of BER by* 0.5 *dB, when spatial correlation is around* 0.9*. Then, equivalence between channel power delay profile (PDP) parameters and parameters extracted from subspace methods in frequency domain is demonstrated. This has the advantage of improving the performance of the channel estimation in frequency domain.*

1. Introduction

Future wireless systems require high data rates. Conventional systems are limited by inter-symbol-interference (ISI) due to frequency selectivity of the wireless channel. By sending information in parallel with larger symbol durations, OFDM systems avoid the ISI significantly [2].

The data rate can be further increased via the exploitation of the MIMO technique. MIMO offers additional parallel channels in spatial domain to boost the data rate. Hence, MIMO-OFDM is a promising combination for the high data requirement of future wireless systems [6].

Coherent demodulation of the transmitted symbols requires accurate channel estimation. MIMO-OFDM channel estimation can performed in frequency and/or time domains. In frequency domain, channels at each OFDM subcarrier are estimated [2]. In time domain estimation, the unknowns are the channel length, tap delays, and their corresponding coefficients [8].

Channel estimation methods can be improved by using the side information [7]. In OFDM systems, the side information can be the correlation due to the time evolution of the channel, the correlation between channel taps and OFDM subcarriers [2], [7]. Many studies exploited time and frequency domain correlation to get the advantages of both domains [6]. With MIMO, correlation from spatial domain exists. Spatial correlation arise due to close antenna spacings and poor scattering environments. Since channel impulse response between co-located transmitter antennas experience the same delay to a receiver point, channel taps corresponding to the same time delay will have the spatial correlation, which can be utilized via some filtering as it is done with the frequency/time domain correlation.

In this paper, first, spatial filtering is explored to further refine the estimates in frequency domain. Then, a relation between channel power delay profile (PDP) parameters and parameters obtained via subspace methods in frequency domain is derived. This relationship has potential applications on estimating the PDP parameters more efficiently as well as improving the performance of frequency domain MIMO-OFDM channel estimation. Potential improvements are the development of less complexity low-rank frequency domain MIMO-OFDM channel estimation techniques, and more accurate estimation of frequency domain channel correlation matrix due to the availability of many MIMO channels with the same PDP.

2. System Model

The system model is given in Fig.1. A MIMO-OFDM system with N_{tx} transmit and N_{rx} receive antennas is assumed. The system has K subcarriers in an OFDM block, and another K_o subcarriers are added as a guard band, also known as cyclic prefix (CP). The incoming bits are modulated to form $X^i[n,k]$, where i is the indexing for transmit antenna, n is the OFDM symbol number, and k is the subcarrier. For each modulated signal, an Inverse Fast Fourier Transform (IFFT) of size K is performed, and the CP is added to mitigate for the residual ISI due to previous OFDM symbol. After parallel-to-serial (P/S) conversion, signal is

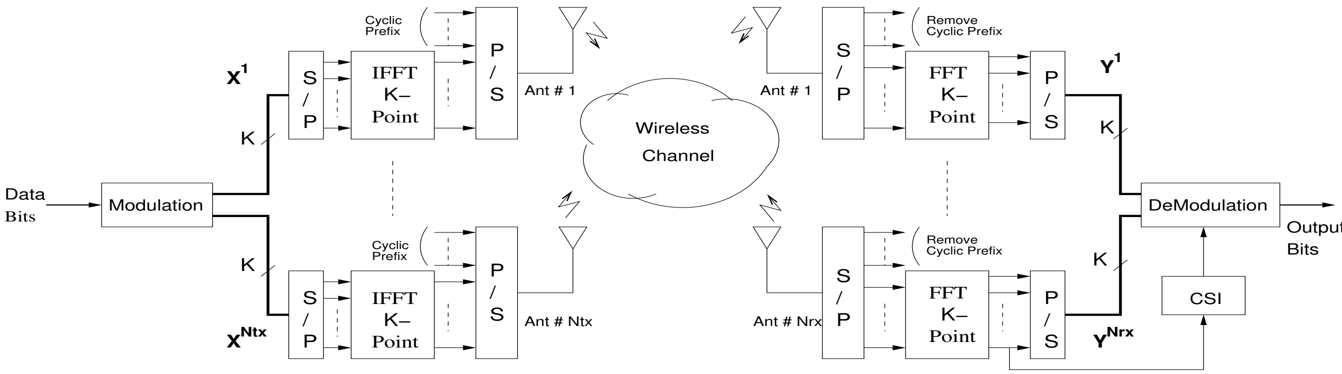

Figure 1. MIMO-OFDM transceiver model.

transmitted from the corresponding antenna. The channel between each transmitter/receiver pair is modeled as multi-tap channel, whose delay characteristic is assumed to be the same for all available channels. The channel is expressed as

$$h(t,\tau) = \sum_{l=0}^{L-1} \alpha_l(t)\delta(\tau - \tau_l), \quad (1)$$

where L is the number of taps, α_l is the l^{th} complex path gain, and τ_l is the corresponding path delay. The path gains are wide-sense stationary (WSS) complex Gaussian processes generated via the method in [5]. It is assumed that different paths are uncorrelated. The frequency response of the channel is given by,

$$H(t,f) = \int_{-\infty}^{+\infty} h(t,\tau)e^{-j2\pi f\tau}d\tau. \quad (2)$$

For OFDM systems with proper CP and timing, the channel frequency response can be written as [4],

$$H[n,k] = H(nT_f, k\Delta f) = \sum_{l=0}^{L-1} h[n,l]F_K^{kl}, \quad (3)$$

where $h[n,l] = h(nT_f, kT_s/K)$, $F_K = e^{-j2\pi/K}$, T_f is the block length, Δf is the subcarrier spacing, and T_s is the symbol duration.

At the receiver side, first serial-to-parallel (S/P) conversion is performed and the CP is removed. After FFT operation, the received signal can be expressed as,

$$Y^q[n,k] = \sum_{i=1}^{N_{tx}} H^i[n,k]X^i[n,k] + w^q[n,k], \quad (4)$$

where q denotes the receiver antenna indexing and w is the additive white Gaussian noise (AWGN).

3. Spatial Filtering

Comb type pilots are commonly used in MIMO-OFDM channel estimation [7]. These have the advantage that the received signal at a given subcarrier is free-of-interference since other transmit antennas are silent when one of the antenna transmits at a given subcarrier. The channel estimation at the silent subcarriers are obtained through interpolation techniques [1]. In this paper, the estimation is performed over 2-OFDM symbols, over which the channel is assumed to be constant. For a better interpolation, the pilots are cyclically shifted by the amount of $N_{tx}/2$. With this pilot scheme, the summation in the expression (4) can be removed. Moreover, the q index can be dropped by concentrating only to a single receive antenna.

3.1. Estimation Method

The pilot grid in frequency and spatial axes puts the problem in a two-dimensional (2-D) domain. By stacking up all the equations, and putting the transmit signals to the diagonal element of a square matrix $(K \times K)$, the received signal, $\mathbf{Y}(K \times 1)$, can be written as

$$\mathbf{Y} = diag(\mathbf{X})\mathbf{H} + \mathbf{W} \quad (5)$$

where $diag(\mathbf{X})(K \times K)$ are the transmitted symbols, $\mathbf{H}(K \times 1)$ represents the unknown channel gains, and $\mathbf{W}(K \times 1)$ is the uncorrelated AWGN with zero mean and σ_n^2 variance. The linear minimum mean square error (LMMSE) estimation of the above equation is the well-known Wiener filtering, and is given by

$$\hat{\mathbf{H}} = \mathbf{R_{HY}}\mathbf{R_{YY}^{-1}}\mathbf{Y} \quad (6)$$

and the mean square error (MSE) is given by,

$$MSE = K - Tr\left[\mathbf{R}_{HY}^H \mathbf{R}_{YY}^{-1} \mathbf{R}_{YH}\right], \quad (7)$$

where $\mathbf{R}_{PQ}$ is the correlation between the variables P and Q, $Tr(.)$ denotes the trace of a matrix, and $(.)^H$ represents the conjugate-transpose. The MSE expression given above is smaller with the larger the two-norm of $\mathbf{R}_{HY}$ [3]. In case of no-spatial correlation, $\mathbf{R}_{HY}$ will be block-diagonal, and the elements representing correlation between subcarriers across antennas will be zero. With spatial correlation, these elements will be non-zero. Hence, it is expected that the use of 2-D Wiener filtering will give better results. Since Wiener filtering in 2-D is computationally complex [3], two cascaded Wiener filters are preferred.

The Wiener filtering requires initial estimates of a set of subcarriers that can be obtained via least squares (LS) estimation, $\hat{\mathbf{H}}_{LS}$. Let $\mathbf{P}^i$ $(1 \times N_p)$ be the set of pilot subcarriers of antenna i. The received signal at subcarriers $p \in \mathbf{P}^i$ is

$$\mathbf{Y_p} = diag(\mathbf{X^i})_\mathbf{p}\mathbf{H^i_p} + \mathbf{W_p}. \tag{8}$$

Then, the LMMSE estimation can be expressed as [2]

$$\hat{\mathbf{H}}^i_{lmmse} = \mathbf{R}_{HH_p}(\mathbf{R}_{H_pH_p} + \beta/SNR)^{-1}\hat{\mathbf{H}}^i_{LS}, \tag{9}$$

where $\mathbf{R}_{HH_p}$ is the frequency domain channel correlation matrix between all subcarriers and pilot subcarriers, $\mathbf{R}_{H_pH_p}$ is the correlation between pilot subcarriers, SNR is the signal-to-noise ratio, and $\beta = E\{|x_k|^2\}E\{1/|x_k|^2\}$.

3.2. Channel Estimation in Spatial Domain

It is essential that the spatial correlation between antennas be translated to the subcarriers correlation in spatial domain. The time domain channel is described in terms of taps, and it is further assumed that the channel delays are the same for all available channels between transmit/receive pair. This is not an unrealistic assumption since the transmitter or receiver antennas are co-located.

Due to the existence of spatial correlation, complex channel gains, $\alpha^i_l(t)$s in (1) are spatially correlated across antennas. For the sake of simplicity, only transmitter antennas are assumed to be spatially correlated. Let the spatial correlation between i^{th} and j^{th} transmit antennas be ρ_{ij}. It can be shown that OFDM subcarriers across antennas will have the same spatial correlation, i.e. $E\left\{H^i[k](H^j[k])^H\right\} = \rho_{ij}$, assuming that the taps for a given channel are temporally uncorrelated. Hence, spatial correlation can be exploited in frequency domain channel estimates without transforming the estimates to the time domain. LMMSE estimation in spatial domain can then be applied to each subcarrier. For a given subcarrier,

$$\mathbf{Z}^k = \mathbf{R}_s(\mathbf{R}_s + \beta/SNR)^{-1}\hat{\mathbf{H}}^k_{lmmse} \tag{10}$$

where $\mathbf{Z}^k$ is the channel estimates for the k^{th} subcarrier across spatial domain, $\mathbf{R}_s$ is the spatial correlation matrix, and $\mathbf{H}^k_{lmmse}$ is the frequency domain LMMSE channel estimate for the k^{th} subcarrier.

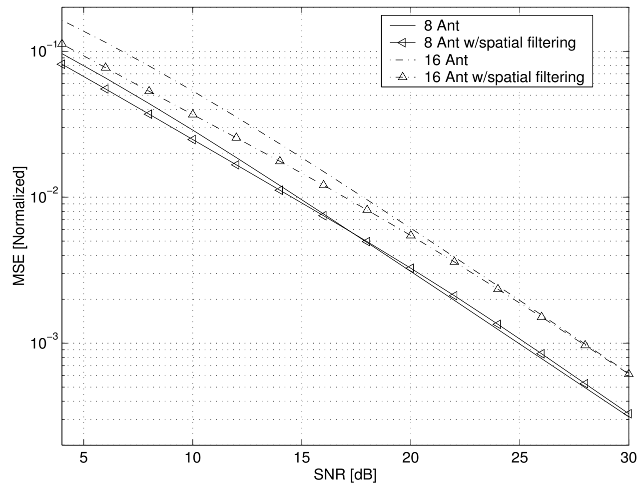

Figure 2. MSE over 2-OFDM symbols.

4. Equivalence Principle

In MIMO-OFDM, the frequency domain estimation technique has to carry the matrix inversion of large matrices (see. Eq. 9). Low-rank approximation methods utilizing the singular value decomposition (SVD) of frequency domain channel correlation matrix are available to simplify (9) [2]. Here, it will be shown that unitary matrices obtained from SVD operation are the unitary FFT matrices, and the singular values are related to the PDP samples via FFT size. This relationship is given below.

The channel frequency response is the FFT of the impulse response, i.e., $\mathbf{H} = \mathbf{Fh}$, where the entries of $\mathbf{F}$ are given by F^{kl}_K. The autocorrelation of the frequency domain channel is given by $E\{\mathbf{HH}^H\}$. Hence,

$$E\left\{\mathbf{HH}^H\right\} = E\left\{\mathbf{Fh}(\mathbf{Fh})^H\right\}. \tag{11}$$

Denoting $\mathbf{R}_{HH} = E\{\mathbf{HH}^H\}$ and $\mathbf{R}_{hh} = E\{\mathbf{hh}^H\}$

$$\mathbf{R}_{HH} = \mathbf{FR}_{hh}\mathbf{F}^H \tag{12}$$

Since $\mathbf{F}$ is a full-rank matrix, the rank or the number of non-zero singular values of $\mathbf{R}_{HH}$ are determined by the rank of $\mathbf{R}_{hh}$. If there are only L number of uncorrelated channel taps, then the rank of $\mathbf{R}_{hh}$ is L. Consider the SVD of the channel correlation matrix. It can be expressed as,

$$SVD\{\mathbf{R}_{HH}\} = \mathbf{U\Sigma V^H}, \tag{13}$$

where $\mathbf{U}$ and $\mathbf{V}$ are orthonormal basis matrices, and $\mathbf{\Sigma}$ is a diagonal matrix with the singular values. Since the rank of $\mathbf{R_{HH}}$ is L, there are only L number of non-zero singular values. With $\mathbf{R_{HH}}$ being a $K \times K$ matrix, the size of $\mathbf{U}$ and $\mathbf{V^H}$ are both $K \times K$. Hence (12) can be written as,

$$\begin{aligned} \mathbf{U\Sigma V^H} &= \mathbf{F R}_{hh} \mathbf{F}^H && (14) \\ &= \mathbf{F_u} K \mathbf{R}_{hh} \mathbf{F_u}^H, && (15) \end{aligned}$$

where F_u denotes the normalized FFT matrix, i.e. each column has a unity norm. Hence

$$\begin{aligned} \mathbf{U} &= \pm \mathbf{F}_u = \mathbf{V} && (16) \\ \mathbf{\Sigma} &= K \mathbf{R}_{hh}, && (17) \end{aligned}$$

provided that the power delay profile is non-sparse. In a sparse channel, care must be taken in the SVD process. Singular values from SVD do not give the information about the tap order. However, since $\mathbf{U} = \mathbf{F}_u$, the time delay information can be extracted from $\mathbf{U}$ matrix. With this equivalence principle, less complexity low-rank methods in frequency domain can be developed using unitary FFT matrices, which is an ongoing research.

5. Results

A QAM-4 modulated MIMO-OFDM system with 64 subcarriers is considered. An exponential channel delay profile with $\tau_{rms} = 2\mu sec$ is assumed. The channel length is taken to be 5, so is the length of CP. A circular arrays is considered with a pre-defined correlation values for transmit side. The MSE of the estimated channels are obtained and is shown in Fig. 2. As can be seen the performance of the estimation is improved with spatial filtering. BER results are shown in Fig. 3. It is seen in the zoomed graph that with 0.9 correlation, a gain of 0.5 dB is possible in 16 transmit antenna system. Although not shown here, with correlation values less than 0.85, no gain is available. Besides, the gain reduces as the number of antennas decreases.

6. Conclusion and Discussion

The effect of spatial filtering in the channel estimation of MIMO-OFDM systems is investigated. First, frequency domain correlation is utilized through LMMSE filtering in frequency domain. This is then followed by spatial domain LMMSE filtering. Simulation results show that additional spatial filtering provides 0.5 dB gain in BER performance of a MIMO-OFDM system with 16 transmit antennas and 0.9 spatial correlation. Higher gain is possible with more antennas and spatial correlation. Moreover, equivalence between channel PDP parameters and the parameters from frequency domain subspace methods is presented. Since in MIMO-OFDM systems all available channels have the same PDP, correlation matrices needed in frequency domain LMMSE filtering can be obtained more accurately in a shorter time,

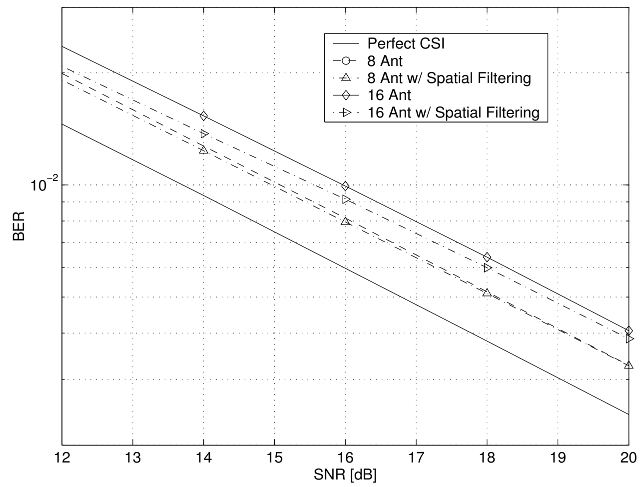

Figure 3. MSE vs. spatial correlation.

and the corresponding low-rank channel estimation algorithms can be further simplified using the equivalence principle. Hence, MIMO-OFDM channel estimation can be performed only in frequency domain.

References

[1] A. Dowler and A. Nix. Performance evaluation of channel estimation techniques in a multiple antenna OFDM system. In *Proc. IEEE Vehic. Techn. Conf.*, volume 1, pages 1–4, Orlando, FL, Oct. 2003.

[2] O. Edfors, M. Sandell, J.-J. van de Beek, S. K. Wilson, and P. O. Borjesson. OFDM channel estimation by singular value decomposition. *IEEE Trans. Commun.*, 46(7):931–939, July 1998.

[3] Z. Guo and W. Zhu. 2-D channel estimation for OFDM/SDMA. In *Proc. IEEE World Wireless Congress*, San Francisco, CA, May 2002.

[4] Y. Li, J. H. Winters, and N. R. Sollenberger. MIMO-OFDM for wireless communications, signal detection with enhanced channel estimation. *IEEE Trans. Commun.*, 50(9):1471–1477, Sept. 2002.

[5] M. K. Özdemir, H. Arslan, and E. Arvas. A narrowband MIMO channel model with 3-D multipath scattering. In *Proc. IEEE Int. Conf. Commun.*, Paris, France, June 2004.

[6] J. Siew, R. Piechocki, A. Nix, and S. Armour. A channel estimation method for MIMO-OFDM systems. In *Proc. London Commun. Symp.*, pages 1–4, London, England, Sept. 2002.

[7] L. Tong, B. M. Sadler, and M. Dong. Pilot assisted wireless transmissions. IEEE Signal Processing Mag. (submitted), July 2003.

[8] B. Yang, K. B. Letaief, R. S. Cheng, and Z. Cao. Channel estimation for OFDM transmission in multipath fading channels based on parametric channel modeling. *IEEE Trans. Commun.*, 49(3):467–479, Mar. 2001.