Noise Power and SNR Estimation for OFDM Based Wireless Communication Systems

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ABSTRACT

In this paper, noise power and signal-to-noise ratio (SNR) estimations for OFDM based wireless communication systems are studied. Noise power estimation, which takes into account the color and variation of noise statistics over OFDM sub-carriers into account, is considered. Instead of averaging the instantaneous noise samples estimates over all of the OFDM sub-carriers, dividing the total number of sub-carriers into several sub-groups and averaging the sub-carriers separately within each sub-group is proposed. Also, these averaged estimates over each sub-group are further averaged across several OFDM symbols in time to enhance the performance of the noise power estimates. The proposed solution provides many local estimates, allowing tracking of the variation of the noise statistics across OFDM sub-carriers, which is particularly of use in sub-band adaptive modulation OFDM systems. The performance of the proposed method is evaluated via computer simulations. It is observed that the proposed solution can estimate local statistics of the noise power when the noise is colored. When the noise is white, the proposed algorithm works as good as the conventional noise power estimation schemes, showing the robustness of the proposed method.

KEY WORDS

SNR, Noise variance estimation, Adaptive modulation

1 Introduction

Signal-to-noise ratio (SNR) is broadly defined as the ratio of the desired signal power to the noise power. SNR estimation indicates the reliability of the link between the transmitter and receiver. In adaptive system design, SNR estimation is commonly used for measuring the quality of the channel. Then, the system parameters are changed adaptively based on this measurement. For example, if the measured channel quality is low, the transmitter adds some redundancy or complexity to the information bits (more powerful coding), or reduces the modulation level (better Euclidean distance), or increases the spreading rate (longer spreading code) for lower data rate transmission. Therefore, instead of fixed information rate for all levels of channel quality, variable rates of information transfer can be used to maximize system resource utilization with high quality of user experience.

One particular interest for SNR estimation is to use it for Adaptive Orthogonal Frequency Division Multiplexing (AOFDM) based wireless communication systems. Adaptive modulation that employs different level of modulation for each (or a group) of sub-carriers depends strongly on accurate estimation of SNR value [1]-[3]. There are many other applications that can exploit SNR information, like channel estimation through interpolation and optimal soft information generation for high performance decoding algorithms [4, 5].

In previous SNR estimation techniques, the SNR measurement is considered as an indication of long-term fading statistics due to shadowing and log-normal fading. This long-term SNR estimate is often calculated using regularly transmitted training (or pilot) sequences. Instead of using training sequences, the data symbols can also be used for this purpose. For example, Balachandran, who uses SNR information as a channel quality indicator for rate adaptation, exploits the cumulative Euclidean metric corresponding to the decoded trellis path for channel quality information [6]. Jacobsmeyer [7] describes another method for channel quality measurement. He proposes the use of the difference between the maximum likelihood decoder metrics for the best path and the second best path [7]. In a sense, he uses some sort of soft information for channel quality indication. But, this approach does not provide any information about the strength of the interferer or the desired signal. There are several other SNR measurement techniques which can be found in [8] and reference listed therein.

In many SNR estimation techniques, the noise is assumed to be white and Gaussian distributed. However, in wireless communication systems, noise is often caused by a strong interferer, which is colored in nature. Color of the noise is defined as the variation in power spectral density in the frequency domain. Of particular interest is OFDM based multi-carrier modulation systems, where the channel bandwidth is wide and the interference is not constant over the whole band. It is very likely that there is variation of spectral content over the OFDM sub-carriers i.e. some part of the spectrum is affected more by the interferer than the other parts. Figure 1 shows OFDM frequency spectrum and two types of noise over this spectrum, colored and white.

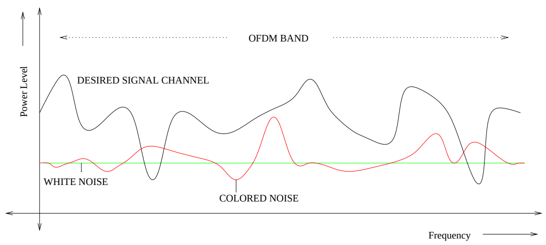

Figure 1. Representation of OFDM frequency channel response and noise spectrum. Spectrum for both white and colored noise is shown.

In many new generation wireless communication systems, coherent detection is employed, which requires estimation of channel parameters. These channel parameter estimates can also be used to calculate the signal power. Therefore, SNR estimation introduces only an additional estimation of noise power. Hence, in this paper we focus more on estimation of noise power, and assume that the signal power can be estimated from the channel estimates. Most commonly used approach for noise power estimation in OFDM systems is based on finding the difference between the noisy received sample in frequency domain and the best hypothesis of the noiseless received sample [1, 9]. Calculation of the received sample hypothesis requires channel state information for each carrier. As mentioned above, the previous approaches estimate long term noise power values, assuming that the noise is white and Gaussian distributed. In order to get the long term estimates, the instantaneous SNR estimates are averaged over the whole OFDM band by taking the mean of all the estimates over all the sub-carriers.

In this paper, the assumption of the noise to be white is removed. Also, variation of the noise power across OFDM sub-carriers is allowed. Therefore, the proposed approach estimates both local (within smaller sets of sub-carriers) and global (over all sub-carriers) SNR values. The short term local estimates calculate the noise (or interference) power variation across OFDM sub-carriers. These estimates are specifically very useful for adaptive modulation, and optimal soft value calculation for improving channel decoder performance. However, these estimates are not reliable, requiring averaging. In this paper, averaging across several OFDM symbols as well as averaging across OFDM sub-carriers are proposed. The proposed approach is evaluated through computer simulations. As will be shown in the performance section, the proposed approach can estimate both local and global statistics of the noise very well.

2 System Model

An OFDM based system model is used. Time domain samples of an OFDM symbol can be obtained from frequency domain symbols as

$$\begin{aligned} x_m(n) &= IDFT\{S_m(k)\} \\ &= \sum_{k=0}^{N-1} S_m(k) e^{j2\pi nk/N} \qquad 0 \leq n \leq N-1 \,, \end{aligned}$$

where $S_m(k)$ is the symbol that is transmitted on k-th sub-carrier of the m-th OFDM symbol, and N is the number of sub-carriers. After the addition of cyclic prefix and D/A conversion, the signal is passed through the mobile radio channel. Assuming a wide-sense stationary and uncorrelated scattering (WSSUS) channel, the channel $H(f,t)$ can be characterized for all time and all frequencies by the two-dimensional spaced-frequency, spaced-time correlation function

$$\phi(\Delta f, \Delta t) = E\{H^*(f,t)H(f+\Delta f, t+\Delta t)\} \,. \qquad (1)$$

In this paper, we assume the channel to be constant over an OFDM symbol, but time-varying across OFDM symbols, which is a reasonable assumption for low and medium mobility.

At the receiver, the signal is received along with noise. The noise power is assumed to be varying across OFDM sub-carriers as well as in time. After synchronization, down sampling, and removing the cyclic prefix, the simplified received baseband model of the samples can be formulated as

$$y_m(n) = \sum_{l=0}^{L-1} x_m(n-l) h_m(l) + z_m(n) \,, \qquad (2)$$

where L is the number of channel taps, $z_m(n)$ is the noise sample which is combination of white Gaussian noise and colored interference source, and the time domain CIR, $h_m(l)$, over an OFDM symbol is given as time-invariant linear filter. After taking DFT of the OFDM symbols, the received samples in frequency domain can be shown as

$$\begin{aligned} Y_m(k) &= DFT\{y_m(n)\} \\ Y_m(k) &= S_m(k) H_m(k) + Z_m(k) \,, \end{aligned} \qquad (3)$$

where $H_m(k)$ and $Z_m(k)$ are DFT of $h_m(l)$ and $z_m(n)$, respectively.

Note that, in this paper, the noise is not assumed to be white. In practical wireless communication systems, often the received signal is impaired by dominant interference sources. For example, in cellular systems, the dominant interference source can be a co-channel or an adjacent channel interferer. In WLAN systems, this can be a Bluetooth interference (like in 802.11g), or due to any other colored dominant noise source (like baby monitors, cordless phones etc.)

3 Estimation of noise power

The proposed noise power estimation is performed in frequency domain. As in [1, 9], the instantaneous noise estimate for each OFDM carrier is calculated by finding difference between noisy received sample and the best hypothesis of the noiseless received signal

$$E_m(k) = |Y_m(k) - \hat{S}_m(k)\hat{H}_m(k)|^2 \qquad (4)$$

where $\hat{S}_m(k)$ is the best hypothesis of the received symbol and $\hat{H}_m(k)$ is the channel estimate for the kth carrier of the mth OFDM symbol. For noiseless channel estimates and for correctly detected symbols, the above equation will provide the absolute square of the exact instantaneous noise samples. However, the channel estimation error and incorrect decisions will bias the noise estimates. The problem with incorrect symbol estimates can be resolved by estimating noise samples using only known data (training symbols), or by finding the error after decoding and re-encoding the detected symbols. Using the decoded information improves performance as the decoder corrects the incorrect decisions.

The channel estimates in frequency domain can be obtained using OFDM training symbols, or by transmitting regularly spaced pilot symbols in between the data symbols and by employing frequency domain interpolation. In this paper, we assume transmission of training OFDM symbols. Using the knowledge of the training symbols, channel frequency response can be estimated as

$$\begin{aligned} \hat{H}_m(k) &= \frac{Y_m(k)}{S_m(k)} \\ \hat{H}_m(k) &= H_m(k) + w_m(k) , \end{aligned} \qquad (5)$$

where $w_m(k)$ is the channel estimation error.

In conventional noise power estimation algorithms [1, 9], the absolute square of the instantaneous noise samples are averaged over all OFDM sub-carriers, providing an averaged noise power estimate. The conventional approaches assume the noise to be white Gaussian distribution and estimates a single noise variance (power) for all the OFDM sub-carriers. Therefore, these approaches do not provide any information about the variation of noise within the transmission bandwidth.

In this paper, we divide the whole band (i.e. the total number of sub-carriers) into sub-bands (i.e. to a set of sub-carriers). If the number of sub-carriers in each sub-band is K, then the number of sub-bands will be N/K. Then, the absolute square of the instantaneous noise estimates in each sub-band are averaged,

$$\hat{E}_m(s) = \frac{1}{K}\sum_{l=1}^{K} E_m(l) , \qquad 1 \le s \le N/K \qquad (6)$$

where $\hat{E}_m(s)$ is the estimated power in the s^{th} sub-band. The size of K depends on the color of the noise. If the noise is completely white, then it is desired that averaging be done across all the available OFDM sub-carriers, i.e. to have K equal to N. Note that if the noise is white and Gaussian distributed, $\hat{E}_m(s)$ has a chi-square distribution with K degrees of freedom, and with mean σ_m^2 and variance $\frac{2(\sigma_m^2)^2}{K}$, where $\hat{\sigma}_m^2$ is the variance of the white noise. The variance of $E_m(s)$ is the mean-square-error of the noise power estimator. Therefore, increasing the number of samples over which averaging is done yields a lower mean-square-error in the case of white noise, but the same does not apply for colored noise. The averaged noise estimates over each sub-band are further averaged across several OFDM symbols (averaging in time). A sliding window averaging is used for time domain averaging,

$$E(s) = \frac{1}{M}\sum_{m=1}^{M} \hat{E}_m(s) . \qquad (7)$$

Therefore, instead of having one noise power estimate for all the sub-carriers, we obtain N/K estimates, where each estimate represents the noise power estimate over each sub-band. The frequency and time averaging of the instantaneous noise samples are shown in Figure 2. The averaging window in time depends on the variation of the sub-band noise power in time. If the noise statistics vary rapidly, then it is desired to choose smaller window size in order to be able to track the variations of the noise power. On the other hand, if the noise statistics change slowly, it is desired to have larger window size to allow better averaging of the sub-band noise values.

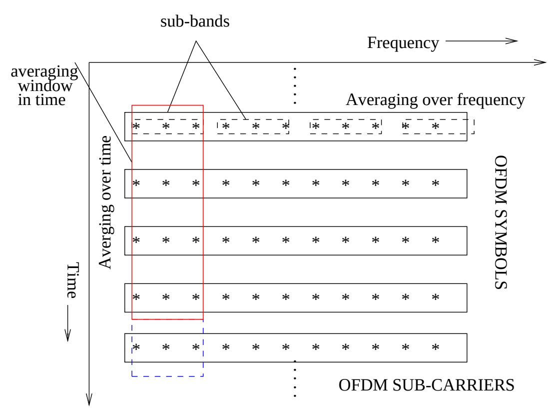

Figure 2. Short and long term noise power estimation through averaging in both frequency and time. In this example, each sub-band includes 3 sub-carriers (i.e. $K = 3$). Also, time domain averaging window size is shown as 4.

4 Performance Results

The performance results are obtained through computer simulations. An OFDM system with 64 sub-carriers is considered. For the colored noise, a co-channel interference source, which is modelled as another OFDM symbol, is used. The channels for the desired and interfering signals are uncorrelated, varying in frequency (frequency selective) and in time (time selective). The channel taps are obtained using modified Jake's fading model [10]. Frequency variation depends on the rms delay spread of the channels, and time variation depends on the Doppler spread. Carrier frequency of 5.2 GHz, and OFDM symbol duration of 4 μsec are considered in the simulations. Additive white noise is included apart from colored noise, and performance evaluations are made for a white and color noise dominated environments, which are created by varying their respective ratio.

Figure 3 compares the conventional and proposed noise power estimates over one realization of the channel. The average signal power is normalized to 0 dB and signal-to-interference ratio of 7 dB is used in this figure, i.e. the average noise power is -7 dB. As described before, the conventional noise power estimate measures only a single value over the whole frequency band. This value is the averaged noise power over the whole band. As can be seen, the conventional estimate works well in measuring the average noise power. The proposed scheme measures both local and global noise power estimates. The total band is divided into 16 groups of sub-bands. In each subgroup, there are 4 sub-carriers. The instantaneous absolute square of the noise values are averaged over each subgroup. Then, these averaged values over each sub-group are further averaged over 50 OFDM symbols (averaging in time domain). The figure shows the averaged noise power estimates over each sub-group. For the reference purpose, the actual noise power values in each sub-group are also given. As can be seen, the proposed algorithm estimates the local noise power very well. In the figure the desired signal power over each sub-group is also given. From these, SNR values over each sub-group can be calculated easily. Notice that the noise power estimate performance depends on the SNR value over each sub-group. When the SNR value is very low, the estimates deviate from the actual noise power value due to incorrect decisions. As described before, the estimates can be improved further by using decoded decisions.

Figure 4 shows the mean-square-error performance of the proposed and conventional algorithms in colored noise. The interference limited scenario as mentioned above is considered with different interference power levels. The mean-squared-error between the actual noise and estimated noise values in each sub-group are calculated and averaged. As can be seen the proposed algorithm performs much better than conventional noise power estimation in terms of finding the local noise power.

Figure 5 shows the mean-square-error performance in white noise. This figure shows the robustness of the proposed algorithm when the noise is not colored. As can be seen, the proposed algorithm works as well as the conventional algorithm which is specifically designed for white noise assumption. Even if the noise is white, the proposed algorithm does not lose performance against conventional scheme. On the other hand, as explained above, if the noise is colored, the proposed algorithm outperforms conventional noise power estimation algorithm.

Figure 6 shows the mean-square-error performance using the proposed method with different window sizes in a pure white noise dominated environment. In white noise dominant case, as the window size increases, there are more samples available for averaging and therefore the estimated noise variance over a larger window size approaches the true value. In colored noise however, the errors depend on the speed, as well as the channel power delay profile.

Figure 7 shows the mean-square-error variation depending on the color of the noise and the frequency selectivity of the channel. The colored interference to white noise ratio I/N is varied from 30 dB, which is highly colored to -30 dB, which is almost white in nature, with different values of rms delay spread (0 μsec, 0.05 μsec and 0.1 μsec) to reflect the frequency selectivity of the channel. It is seen from the plot that a higher window size results in lower error as the color of the noise decreases and resembles white noise.

5 Conclusion

In this paper, noise variance estimation that removes the white noise assumption is described. The proposed algorithm considers a more practical environment where noise is characterized by a non-constant spectral content over the OFDM sub-carriers. This is often the case when the noise is dominated by a strong interference. The autocorrelation of the power spectral density (PSD) reflects the intensity of color. The proposed method can identify the intensity of color of noise. On the other hand, when the noise is not colored, it performs as well as the conventional noise variance estimations which are specifically designed for white noise assumption. The proposed solution can be very useful for adaptive modulation as well as for other adaptive transmitter and receiver algorithms, like optimal soft information calculation, improved channel estimation etc.

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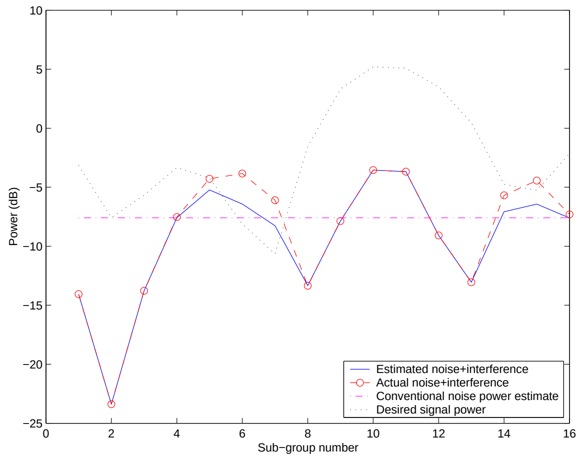

Figure 3. Comparison of conventional and proposed noise power estimation algorithm. A realization of signal and noise powers over the transmission band is shown.

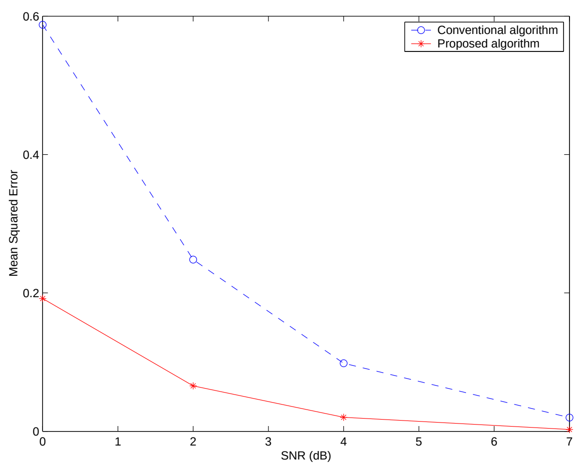

Figure 4. Mean-square-error performance of the conventional and proposed algorithms in colored noise with a window size of 16.

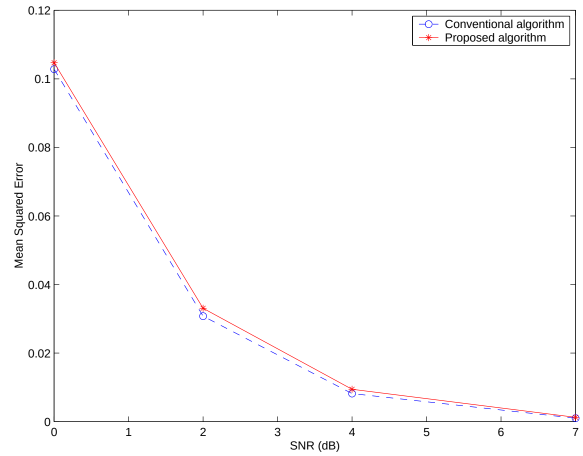

Figure 5. Mean-square-error performance of the conventional and proposed algorithms in white noise.

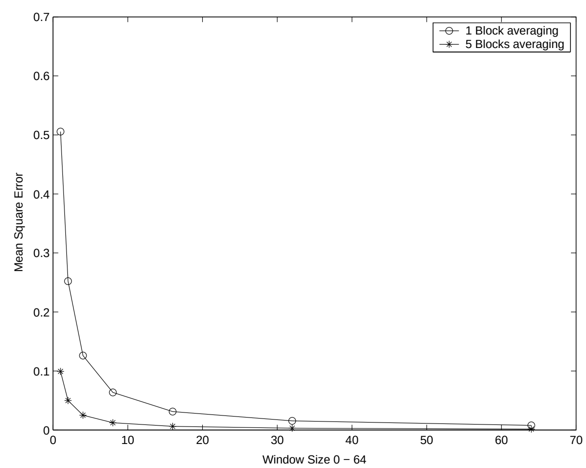

Figure 6. Mean-square-error performance in white noise with different window sizes and time domain averaging over 1 and 5 blocks.

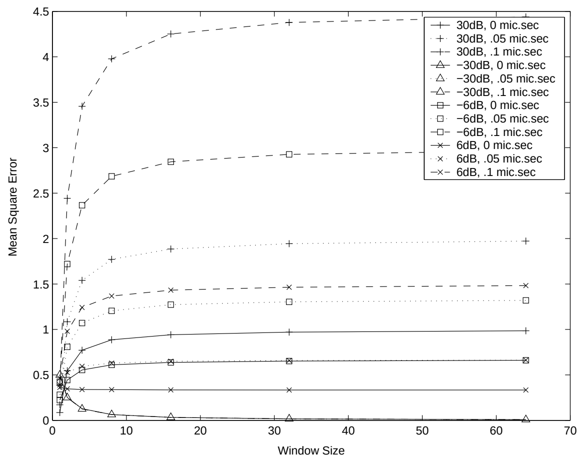

Figure 7. Mean-square-error variation for 1 and 5 block averaging for different color intensities.

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