ICI Cancellation Based Channel Estimation for OFDM Systems

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***Abstract*— Orthogonal Frequency Division Multiplexing (OFDM), which is a multicarrier modulation scheme, splits high data rate information symbols into lower rate parallel data streams and transmits this parallel information on different orthogonal carriers. The loss of orthogonality among subcarriers causes inter-carrier interference (ICI) which affects both channel estimation and detection of OFDM data symbols. Channel estimation error degrades the performance of coherent receiver limiting capacity, data rate and performance of the overall OFDM system. This paper proposes a novel frequency-domain channel estimator which mitigates the effects of ICI by jointly finding the frequency offset and channel frequency response (CFR). Unlike conventional channel estimation techniques, where ICI is treated as part of the noise, the proposed approach considers the effect of frequency offset in estimation of CFR. A binary search algorithm is used to find the present frequency offset and CFR jointly. Variance of the frequency offset estimator and mean-square error (MSE) performances for conventional least-squares (LS) estimator and the proposed method are obtained using computer simulations. The variance of the jointly estimated frequency offset is found to be very close to the Cràmer-Rao lower bound. It is shown that the proposed method outperforms LS channel estimator especially in ICI dominated situations.**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM), which is a multi-carrier modulation scheme, is a strong candidate for future communication systems to achieve high data rates in multipath fading environments. In OFDM, the wide transmission spectrum is divided into narrower bands and data is transmitted in parallel on these narrow bands. Therefore, symbol period is increased by the number of subcarriers, decreasing the effect of inter-symbol interference (ISI). The remaining ISI effect is eliminated by cyclicly extending the signal.

The loss of orthogonality among subcarriers causes inter-carrier interference (ICI). Reason for this loss can be carrier frequency offset [1], Doppler spread [2] or a combination of both. ICI is often modeled as Gaussian noise and affects both channel estimation [3] and detection of the OFDM symbols [4].

Channel estimation is one of the most important elements of wireless receivers that employs coherent demodulation. For OFDM based systems, channel estimation has been studied extensively. Approaches based on least-squares (LS), minimum mean-square (MMSE) [5], and maximum likelihood (ML) [6] estimation are studied by exploiting the training sequences that are transmitted along with the data. The previous channel estimation algorithms treat ICI as part of the additive white Gaussian noise and these algorithms perform poorly when ICI is significant. Linear minimum mean-square error (LMMSE) estimator is analyzed in [7] to suppress the ICI due to mobility (Doppler spread). However, it is shown that non-adaptive LMMSE estimator given in [7] is not capable of reducing ICI and the design of an adaptive LMMSE is relatively difficult since both Doppler profile and noise level need to be known. A channel estimation scheme which uses time-domain filtering to mitigate the ICI effect of time-varying channel is proposed in [8].

This paper proposes a novel channel estimation method that eliminates ICI by jointly finding the frequency offset and channel frequency response (CFR). The proposed method finds channel estimates by hypothesizing different frequency offsets and chooses the best channel estimate using correlation properties of CFR. In the rest of this paper, proposed algorithm will be described briefly and simulation results will be given.

II. System Model

OFDM converts serial data stream into parallel blocks of size N and modulates these blocks using Inverse Discrete Fourier Transform (IDFT). Time domain samples can be calculated as

$$\begin{aligned} x(n) &= IDFT\{X_k\} \\ &= \sum_{k=0}^{N-1} X_k e^{j2\pi nk/N} \qquad 0 \leq n \leq N-1 \ , \end{aligned} \tag{1}$$

where X_k is the symbol transmitted on the kth subcarrier. Time domain signal is cyclicly extended to avoid ISI from previous symbol.

At the receiver, the signal is received along with noise. After synchronization, down sampling, and removal of cyclic prefix, the baseband model of the received frequency domain samples can be written in matrix form as

$$\mathbf{y} = \mathbf{S}_{\epsilon_p}\mathbf{X}\mathbf{h} + \mathbf{n} \ , \tag{2}$$

where $\mathbf{y}$ is the vector of received symbols, $\mathbf{X}$ is a diagonal matrix with the transmitted symbols on its diagonal, $\mathbf{h} = [H_1 \ H_2 \cdots H_N]^T$ is the vector representing the CFR to be estimated, and $\mathbf{n}$ is the additive white Gaussian noise vector with mean zero and variance of σ_n^2. The $N \times N$ matrix, $\mathbf{S}_{\epsilon_p}$, is the interference (crosstalk) matrix that represents the leakage between subcarriers, *i.e.* ICI. If there is no frequency offset, *i.e.* $\epsilon_p = 0$, $\mathbf{S}_{\epsilon_p}$ becomes $\mathbf{S}_0 = \mathbf{I}$, which implies no interference from neighboring subcarriers. If ICI is assumed to be caused only by frequency offset, entries of $\mathbf{S}_{\epsilon_h}$ can be found using the following formula [1]

$$\mathbf{S}_{\epsilon_p}(m,n) = \frac{\sin \pi(m-n+\epsilon_p)}{N \sin \frac{\pi}{N}(m-n+\epsilon_p)} e^{j\pi(m-n+\epsilon_p)} \ , \quad (3)$$

where ϵ_p is the *present* normalized carrier frequency offset (the ratio of the actual frequency offset to the inter-subcarrier spacing).

III. Algorithm Description

The interference matrix $\mathbf{S}_{\epsilon_p}$ is not known to the receiver as it depends on the unknown carrier frequency offset, ϵ_p. In this paper, we will try to match to $\mathbf{S}_{\epsilon_p}$ by $\mathbf{S}_{\epsilon_h}$, where ϵ_h is the hypothesis for the true frequency offset.

The estimate of CFR is obtained by multiplying both sides of (2) with $(\mathbf{S}_{\epsilon_h}\mathbf{X})^{-1}$ as

$$\begin{aligned} (\mathbf{S}_{\epsilon_h}\mathbf{X})^{-1}\mathbf{y} &= (\mathbf{S}_{\epsilon_h}\mathbf{X})^{-1}\mathbf{S}_{\epsilon_p}\mathbf{X}\mathbf{h} + (\mathbf{S}_{\epsilon_h}\mathbf{X})^{-1}\mathbf{n} \\ \mathbf{h}_{\epsilon_h} &= \mathbf{X}^{-1}\mathbf{S}_{\epsilon_h}{}^{-1}\mathbf{S}_{\epsilon_p}\mathbf{X}\mathbf{h} + \mathbf{n}_{\epsilon_h} \ . \end{aligned} \quad (4)$$

The inversion of the matrix $\mathbf{S}_{\epsilon_h}\mathbf{X}$ is simple since the interference matrix $\mathbf{S}_{\epsilon_h}$ is unitary and the data matrix $\mathbf{X}$ is diagonal.

Equation 4 will yield several channel estimates for different frequency offset hypotheses. For the offset hypothesis, ϵ_h, which is closest to the actual frequency offset, ϵ_p, (4) will yield the best estimate of the CFR. For choosing the best hypothesis, channel frequency correlation is used as a decision criteria. In the rest of this section, properties of the interference matrix will be described first. Then, the method for choosing the best hypothesis will be explained followed by the description of the search algorithm to find the best hypothesis.

A. Properties of Interference Matrix

The following properties related to the interference matrix can be derived using (3).

1) $\mathbf{S}^H\mathbf{S} = \mathbf{I}$ **:** Interference matrix is a unitary matrix. Therefore, the inverse of the interference matrix can be calculated easily by taking the conjugate transpose since $\mathbf{S}^{-1} = \mathbf{S}^H$. Note that the superscript *H* represents conjugate transpose.
2) $\mathbf{S}_{\epsilon_1}\mathbf{S}_{\epsilon_2} = \mathbf{S}_{\epsilon_1+\epsilon_2}$ **:** If two interference matrices corresponding to two different frequency offsets are multiplied, another interference matrix corresponding to the sum can be obtained. This property is exploited in the search algorithm.
3) $\mathbf{S}_{-\epsilon} = \mathbf{S}_{\epsilon}^H$ **:** The interference matrix for a negative frequency offset can be obtained from the interference matrix corresponding to a positive frequency offset with the same magnitude by finding the complex transpose.

B. Correlation of Channel Frequency Response

The multiplication of two interference matrices in (4) can be written using the properties of interference matrix as

$$\mathbf{S}_{\epsilon_h}^{-1}\mathbf{S}_{\epsilon_p} = \mathbf{S}_{-\epsilon_h}\mathbf{S}_{\epsilon_p} = \mathbf{S}_{\epsilon_p-\epsilon_h} = \mathbf{S}_{\epsilon_r} \ , \quad (5)$$

where ϵ_r is the difference between the actual frequency offset and frequency offset hypothesis, i.e. *residual* frequency offset.

Using (4) and (5), the estimate of the channel frequency response can be written as

$$\begin{aligned} H_{\epsilon_h}(k) &= \frac{1}{X_k}\sum_{l=1}^{N} X_l H_l \mathbf{S}_{\epsilon_r}(k,l) \\ &+ \frac{1}{X_k}\sum_{l=1}^{N} n_l \mathbf{S}_{\epsilon_h}(k,l) \qquad 1 \le k \le N \ . \end{aligned} \quad (6)$$

Using (6), the frequency correlation of the estimated channel for each OFDM frame can be calculated as

$$\begin{aligned} R_{\mathbf{h}_{\epsilon_h}}(\tau) &= \frac{1}{N-2\tau}\sum_{k=\tau+1}^{N-\tau}\left\{H_{\epsilon_h}(k)H^*_{\epsilon_h}(k-\tau)\right\} \\ &= \frac{1}{N-2\tau}\sum_{k=\tau+1}^{N-\tau}\left\{\frac{1}{X_k}\sum_{l=1}^{N} X_l H_l \mathbf{S}_{\epsilon_\tau}(k,l)\right. \\ &\quad \cdot \frac{1}{X^*_{k-\tau}}\sum_{u=1}^{N} X^*_u H^*_u \mathbf{S}^*_{\epsilon_h}(k-\tau,u) \\ &\quad + \frac{1}{X_k}\sum_{l=1}^{N} n_l \mathbf{S}_{\epsilon_h}(k,l) \\ &\quad \left. \cdot \frac{1}{X^*_{k-\tau}}\sum_{u=1}^{N} n^*_u \mathbf{S}_{\epsilon_h}(k-\tau,u)\right\} \ . \end{aligned} \quad (7)$$

If we assume that the number of subcarriers, N, is large, (7) can be simplified as

$$R_{\mathbf{h}_{\epsilon_h}}(\tau) = \begin{cases} R_h(0) + \frac{\sigma_n^2}{\sigma_s^2} & \tau = 0 \\ R_h(\tau)|\mathbf{S}_{\epsilon_r}(0)|^2 & \tau \neq 0 \end{cases} \quad (8)$$

where $|\mathbf{S}_{\epsilon_r}(0)| = \frac{\sin(\pi\epsilon_r)}{N\sin(\pi\epsilon_r/N)}$ is the magnitude of the diagonal element of interference matrix of residual frequency offset, $\mathbf{S}_{\epsilon_r}$ and σ_s^2 is the variance of the received signal. Note that as residual frequency offset increases, the value of $|\mathbf{S}_{\epsilon_r}(0)|$ decreases, causing the correlation to decrease.

As (8) implies, the correlation of the CFR depends on the residual frequency offset. For a given CFR, channel frequency correlation becomes maximum when the frequency offset hypothesis, ϵ_h, matches to the actual frequency offset. Therefore, the correlation values can be used as a decision criteria for choosing the best hypothesis. For choosing the best hypothesis among several hypotheses, this criteria is used in the search algorithm

According to (8), all the lags of channel correlation can be used for obtaining the best hypothesis. However, as τ increases channel correlation decreases, this degrades the performance of the estimation since the ratio of useful signal power to the noise power becomes smaller. Also, for large τ values, correlations are more noisy since less samples are used to obtain these correlations. Moreover, increasing the number of lags increases the computational complexity as more correlations need to be estimated. Therefore, selection of the number of lags to be used is a design criteria and needs to be further investigated.

In our simulation, only the first correlation value, $R_{\mathbf{h}_{\epsilon_h}}(1)$, is used. However, better results can be obtained by effectively combining the information from other correlation lags.

C. The Search Algorithm

Finding the frequency domain channel for all of the hypotheses and choosing the best hypothesis require enormous computation. The interference matrices for each frequency offset hypothesis should also be precomputed and stored in memory. However, these requirements can be relaxed by employing an optimum search algorithm. Instead of trying all possible frequency offsets, the correct frequency offset is calculated by using binary search algorithm.

The magnitude of the correlation is estimated at the maximum and minimum expected frequency offset values first. If the value at the minimum point is smaller, correct frequency offset is expected to be at the bottom half of the initial interval. Therefore, maximum point is moved to the point between the previous two points and minimum is not changed. If maximum point is smaller, opposite operation is performed. In the second step the same operation is repeated for the new interval. Then, this process is repeated for a predefined number of iterations. Note that CFR needs to be obtained only for one more hypothesis in each iteration after the first iteration. Therefore the total number of CFRs estimated is total number of iterations plus one.

To calculate the CFR for a hypothesis, we do not need to have all the interference matrices. If the interference matrices for ϵ_{max}, $\epsilon_{max}/2$, $\epsilon_{max}/4$, $\epsilon_{max}/8$, $\ldots$ are calculated, where ϵ_{max} is the maximum expected frequency offset, the required interference matrices can be found by using the second property of interference matrix. Moreover, CFR estimates can be calculated without having all of the interference matrices. In (4), received symbols are multiplied by $\mathbf{S}_{\epsilon_h}^{-1}$ and then multiplied with the diagonal matrix $\mathbf{X}^{-1}$. The result of multiplication with $\mathbf{S}_{\epsilon_h}^{-1}$ can be stored and multiplied with $\mathbf{S}_{\epsilon_2}^{-1}$ in the next step to obtain the same result which would be obtained by multiplying $\mathbf{S}_{\epsilon_h+\epsilon_2}^{-1}$.

D. Reduced interference matrix

The interference matrix $\mathbf{S}$ is an $N \times N$ matrix. However, most of the energy is concentrated around the diagonal, *i.e.* interference is mostly due to neighboring subcarriers. The entries away from the diagonal are set to zero in order to decrease the number of multiplications and additions performed during the search algorithm. This will also decrease the memory requirement. The amplitudes of the full and reduced interference matrices are shown in Fig. 1 for normalized frequency offsets of 0.1 and 0.3.

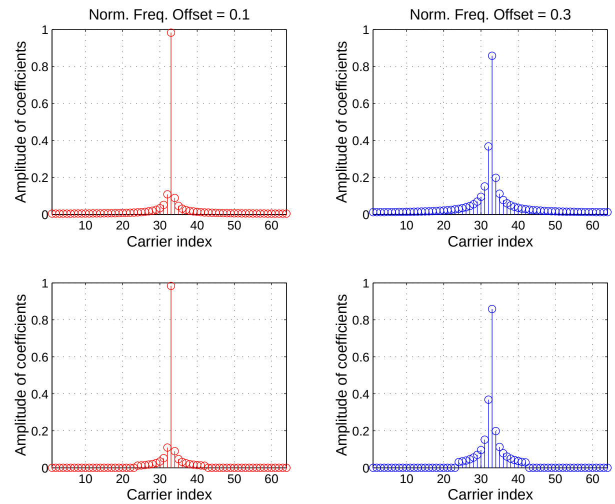

Fig. 1. Magnitudes of both full and reduced matrices for different frequency offsets. Second row shows the reduced matrix. Only one row is shown.

As seen in Fig. 1, the effect of round-off becomes more noticeable as frequency offset increases, since energy will be spread away form the diagonal at high frequency offsets. The gain in computational complexity is more noticeable as the number of subcarriers increases.

IV. Results

Simulation results are obtained in an OFDM based wireless communication system with 64 subcarriers which employs QPSK modulation. A 6-tap symbol-spaced time domain channel impulse response with exponentially decaying power delay profile is used.

Number of iterations for the search algorithm was 8, which means that CFR is estimated for $8+1=9$ different frequency offset hypotheses to find the best CFR.

Fig. 2 shows the variance of the frequency offset estimator as a function of signal-to-noise ratio (SNR). Results for full and reduced interference matrices are shown. Reduced matrix is obtained using the 32 entries of full interference matrix, reducing computational complexity by 50%. The Cràmer-Rao bound [9]

$$CRB(\epsilon) = \frac{1}{2\pi^2}\frac{3(SNR)^{-1}}{N(1-1/N^2)} \qquad (9)$$

is also provided for comparison. As can be seen from this figure, truncating the interference matrix has little effect on the performance.

The frequency range in which the frequency offset is being searched is chosen adaptively depending on the history of the estimated frequency offsets. If the variance of previous frequency offset estimates is small, the range is decreased to increase the performance with the same number of iterations; and if it is large the range is increased in order to be able to track the variations of the frequency offset. Fig. 3 shows the correct and estimated frequency offset values that are obtained by fixing the frequency offset range and changing it adaptively. It can be seen from this figure that the algorithm converges to the correct frequency offset and changing the range adaptively helps tracing the frequency offset.

Mean-square error performances of the proposed and conventional LS estimators are shown in Fig. 4 as a function of SNR, where a normalized frequency offset of 0.05 is used. Obtained channel estimates can be further processed to decrease the mean-square error, however this is out of the scope of this paper.

V. Conclusion

A novel frequency-domain channel estimator which mitigates the effects of ICI by jointly finding the frequency offset and CFR is described in this paper. Unlike conventional channel estimation techniques, where ICI is treated as part of the noise, the proposed approach considers the effect of frequency offset in estimation of CFR. Methods to find the best CFR effectively with low complexity is discussed. It is shown via computer simulations that the proposed method is capable of reducing the effect of ICI on the frequency domain channel estimation.

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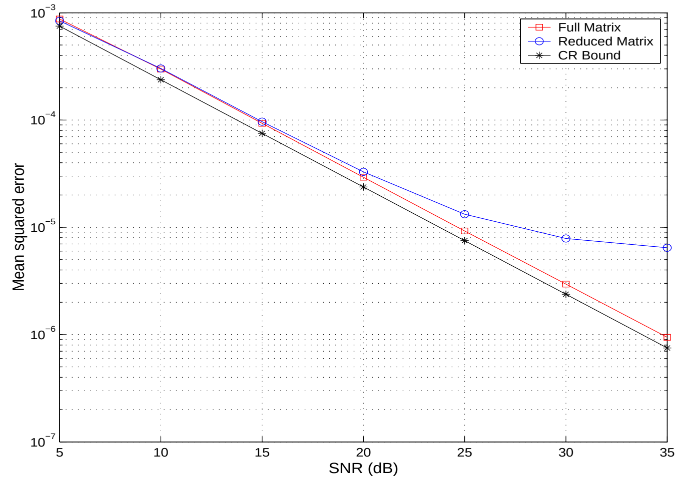

Fig. 2. Variance of the frequency offset estimator. Results obtained by using full and reduced interference matrices and Cràmer-Rao lower bound is shown.

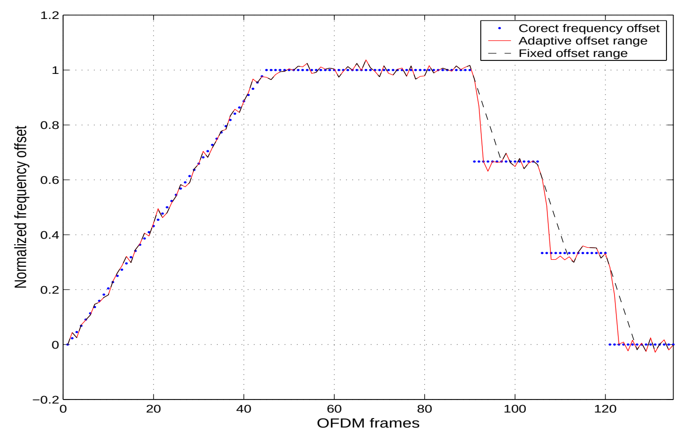

Fig. 3. Estimated and correct (normalized) frequency offset values at 10 dB. Results for adaptive and fixed initial frequency offset ranges.

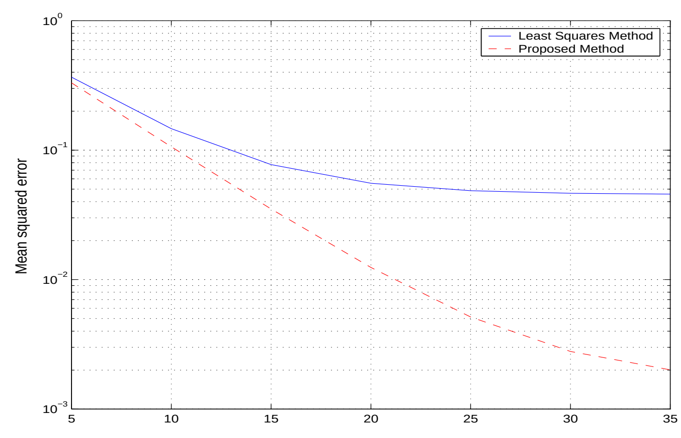

Fig. 4. Mean-square error versus SNR for conventional LS and proposed CFR estimators. Normalized carrier frequency is 0.05.