

# A JOINT MODULATION IDENTIFICATION AND FREQUENCY OFFSET CORRECTION ALGORITHM FOR QAM SYSTEMS

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## ABSTRACT

Modulation identification is one of the most challenging steps for blind receivers. If frequency offset is present in the system, this process becomes even more difficult. The proposed method is based on the estimation of the modulation types for quadrature amplitude modulation (QAM) even before correcting the frequency offset. The modulation order is estimated using the probability density information (pdf) of the amplitude of the received signal. As carrier frequency offset causes a cumulative rotation for every successive sample, amplitude variation of the signal gives very important clues about the order. Using the density information and the number of clusters, the modulation order is estimated. After estimating the modulation type, frequency offset is corrected. If the constellation mapping of the incoming signal is circular, an algorithm defined for phase shift keying (PSK) mappings is used; on the other hand, if it is rectangular, algorithms defined for square constellation mappings are used.

## 1. INTRODUCTION

Modulation identification and carrier frequency offset estimation and correction are very challenging tasks in blind receivers which are becoming very important with the emergence of software defined radio (SDR) applications. There are some methods proposed in the literature to correct the frequency offset; however, the modulation type is assumed to be known for the proposed techniques [1 - 3]. Also for modulation identification, there are some methods presented in the prior art [4, 5]. These algorithms work when there is no frequency offset in the system. So, to the best knowledge of authors', an algorithm does not exist which treats the systems in a fully blind manner, i.e. assuming that the modulation order is not known and frequency offset is present in the system.

Frequency offset is caused by the local oscillator mismatches and wrong carrier synchronization. Each sample is affected by different amount from the frequency offset. The modulation type and order is defined at the transmitter and does not have anything to do with the channel and the

receiver. The receiver only needs to estimate the modulation type and order; however, blind receivers are supposed not to require any knowledge of any parameter on the received signal while demodulating.

In this study, one of the most commonly used modulation types, QAM, is analyzed. The frequency offset is assumed to be less than the symbol rate of the signal and there is no restriction for the phase offset. The bandwidth and the carrier frequency of the incoming signal are estimated and the signal is converted to the baseband and the symbol rate is estimated. After these processes the signal is matched filtered, if required, and downsampled at the optimum sampling instant. The only remaining parameters required for perfect demodulation process are the frequency and phase offsets and the modulation order information.

The proposed method consists of two stages. The first stage of the algorithm determines the modulation order of the incoming signal by inspecting the pdf of the amplitudes. After clustering the amplitudes of the received signal, the algorithm chooses the modulation order and constellation mapping by counting the number of clusters and the density of each cluster. After the modulation identification step, the frequency and phase offsets of the incoming signal are estimated and corrected.

This paper is organized as follows. In Section II, the signal model is given; Section III describes the modulation identification process. The algorithm used to perform compensate the frequency error is presented in Section IV. Section V exhibits the simulation results.

## 2. SIGNAL MODEL

At the receiver, the baseband model of the complex signal can be expressed as the sum of the transmitted signal and noise and is given by,

$$r(n) = s_m(n) \cdot e^{j(\Delta f n T + \Delta \theta)} + v(n) \quad (1)$$

where  $s_m(n)$  is the  $n$ th transmitted data symbol selected from the QAM constellations,  $T$  is the symbol period,  $v(n)$  is the zero-mean complex white Gaussian distributed noise component having variance  $\sigma_v^2$ ,  $\Delta f$  is the carrier frequency offset and  $\Delta \theta$  represents the phase offset. It is assumed that

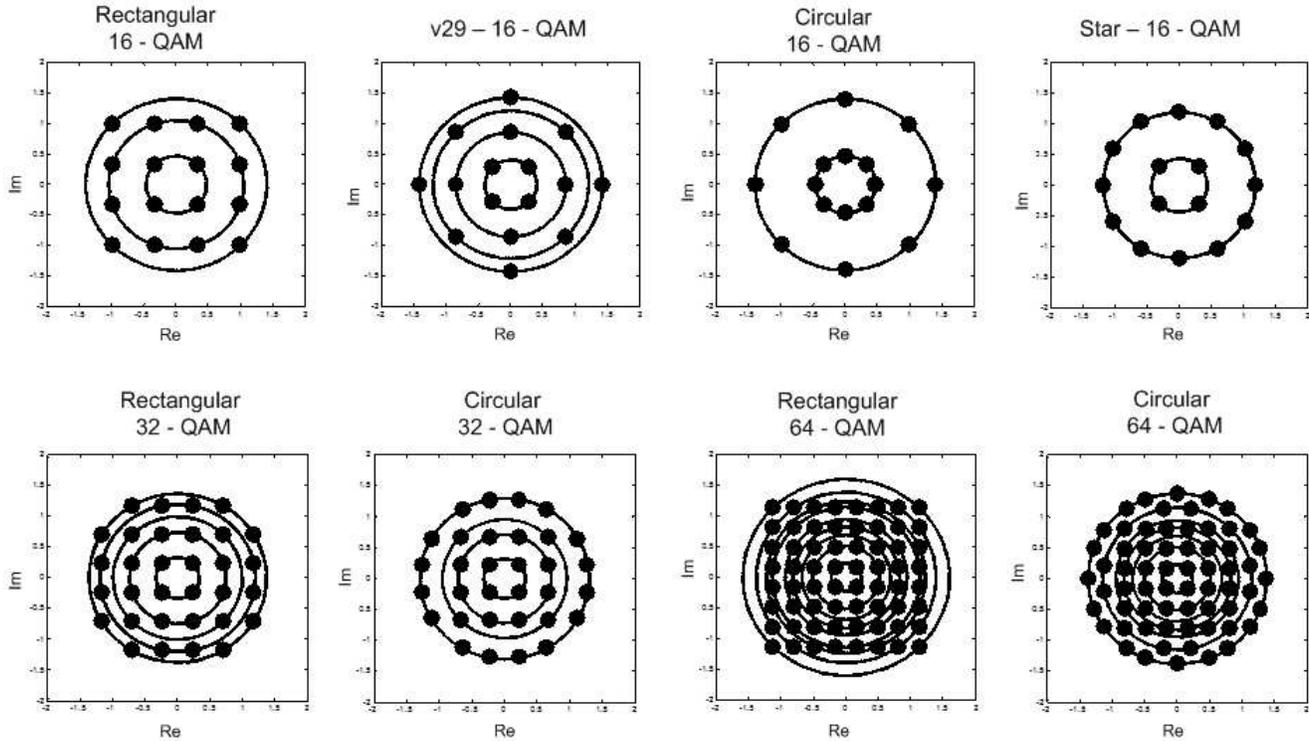


Fig. 1: Different Constellation Mappings for QAM orders of 16, 32 and 64

received signal is matched to the pulse-shaping filter, and the clock synchronization is perfect so that the samples are taken at the optimum sampling points. The carrier frequency offset, the phase offset and the modulation order are the unknown parameters which are to be blindly estimated. The signal-to-noise ratio (SNR) is defined as  $SNR = 10 \log_{10}(1/\sigma_v^2)$ .

### 3. MODULATION IDENTIFICATION

For QAM signals, location of each constellation point depends on amplitude, phase or both; consequently, different orders have different levels of phase and amplitude. For example, in 2<sup>nd</sup> order QAM, there is only one amplitude level and four phase levels, whereas in the rectangular mapping of 4<sup>th</sup> order QAM, there are three amplitude levels and twelve phase levels. Besides, even for the same orders, there are different constellation mappings that have different phase and amplitude levels. If there is no frequency offset present in the system, the modulation order can be detected by inspecting the amplitude and phase levels of the signal. There are modulation identification methods which have already been proposed in the literature such as moment and cumulant based approaches [1]. These estimation methods work well if there is no frequency offset in the system. With the frequency offset present in the system, modulation identification process becomes more difficult. And also, since there are more than one

constellation mapping for some modulation orders, these estimation methods may fail.

In this study, the modulation order is estimated by inspecting the pdf of the amplitude of the received signal. The approach presented in this paper is based on counting the number of rings. Rings are formed when the constellations rotate due to the frequency offset (The circles Fig.1 represents the constellation diagram after frequency offset introduced into the system and the original constellation locations). However, in some cases, just counting the number of rings is not enough. For example, the two different mappings of 16-QAM, i.e. (4, 12) and (8, 8), both have two rings but they have different amount of densities; (k, 3k) and (k, k) respectively. In order to make the discrimination between these modulation types, the density information should be used as well.

The method proposed uses hard k-means algorithm to form the clusters in order to be able to estimate the number of rings. This method has two inputs, one is data set and the other is the number of clusters. According to the number of clusters, the amount of centroids is determined by calculating the shortest distance of each data point to each centroid. Shortest distance method assigns the data points to the closest centroids and, if necessary, updates the new centroid location. This process continues until it reaches to steady state, i.e. when centroid locations stay constant.

In this study, the modulation order having the largest number of rings is the rectangular mapping of 64-QAM,

which has nine rings. Throughout simulations, the cluster number is chosen to be fifteen. In some cases, k-means algorithm assigns two close clusters where both represent the same ring. Because of this property of the algorithm used, the chosen number of cluster to be estimated is larger than the amount needed. Consequently, to be able to get rid of wrong decisions, using more than needed, can be an efficient way to handle this problem. Close clusters can be grouped together to form one cluster and correct number of clusters is obtained and the algorithm's deficiency in generating correct number of clusters is removed [6].

The proposed method starts by finding locations of the fifteen clusters for the received symbol sequence. Since there are more clusters than needed, the close clusters are grouped together and the exact cluster number is obtained. For each cluster, the density is calculated and by using the information of number of clusters then the density information the modulation order is detected. Then the symbol sequence is fed into the frequency and phase offset estimation and correction block.

#### 4. FREQUENCY OFFSET ESTIMATION

With the information obtained, the last step is to choose the optimum frequency offset estimation algorithm to use. The choice is based on the mapping of the constellation diagram.

##### 4.1. Circular Constellation Diagrams

In this section, the focus has been on some orders with circular constellation rings at the outermost edge. Star-16-QAM (4, 12), circular-16-QAM (8, 8), v29-16-QAM (4, 4, 4, 4), circular-32-QAM, and circular-64-QAM are the constellation mappings which will be treated in this section as they can be interpreted as circular mappings.

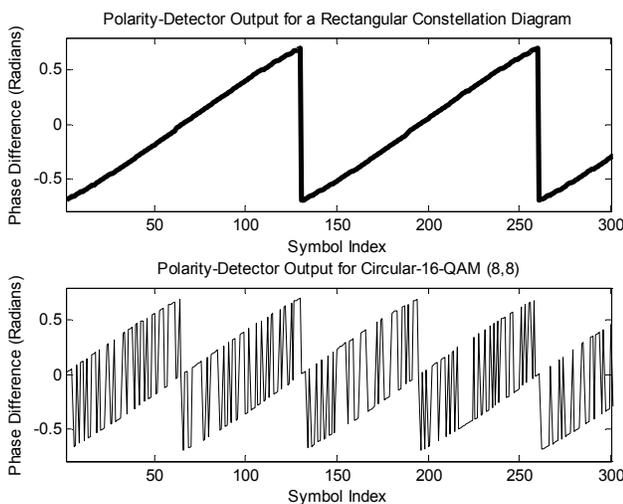


Fig. 2: Output of Polarity-Detector for Different Orders.

To overcome the carrier frequency offset problem, Kim & Choi proposed a phase-locked loop (PLL) based method which is one of the most common solutions to frequency offset problem [4]. This method, known as the polarity detector, is used to find the phase difference of the symbol from the nearest polar;  $\{-3\pi/4, -\pi/4, \pi/4, 3\pi/4\}$ . This process works well for rectangular type of QAM constellations. However, it cannot track the phase difference in circular QAMs like circular-16-QAM (8, 8), and star-16-QAM (4, 12). The output of a polarity detector for a signal with rectangular outermost edge and a circular-16-QAM signal can be seen in Fig. 1. Both signals have positive carrier frequency offset.

Polarity detector is widely used for modulation types with square type of constellation diagrams. If the number of constellation points in the outermost ring exceeds 4, polarity detector would fail at tracking these points which is shown in Fig. 2.

The frequency offset estimation part for circular constellations consists of a threshold value, a phase-error detector, a Stop&Go (S&G) controller and a differentiator [5]. In order to describe the principles of the frequency offset estimator clearly, the phase noise is discarded. The received signal  $r(n)$ , is first multiplied by the output of the differentiator  $\square(n)$  to create a phase compensated signal  $r_c(n)$  and is given by,

$$r_c(n) = r(n) \cdot e^{-j\theta(n)} \quad (2)$$

To be able to take the outermost constellation points, which have the highest SNR values, a threshold value is determined, then the offset estimation algorithm is run. The phase difference is calculated by using  $r_c(n)$  and the decision symbol  $d(n)$ . Decision symbol is the constellation point closest to the received symbol. The closest point is assigned by finding the symbol  $S_m$  that minimizes the Euclidean distance,

$$D(r_c, S_m) = \sum_{k=1}^N (r_{ck} - S_{mk})^2 = |r_c|^2 - 2 \cdot r_c \cdot S_m + |S_m|^2 \quad (3)$$

$D(r_c, S_m)$ ,  $m = 1, 2, \dots$ , is referred as the distance metric. As the minimum distance detection criterion defines, finding the closest constellation point  $S_m$  is based on maximum-likelihood (ML) criterion.  $S_{mk}$  is derived from the previous part of the algorithm which gives the modulation order information. Minimum value of  $D(r_c, S_m)$  defines the  $m^{th}$  value distance metric which will be the decision symbol  $d(n)$  and used for calculating the phase difference. This process is repeated for every successive symbol. In the phase detector,  $r_c(n)$  signal is processed with the decision symbol  $d(n)$ . The phase difference between  $r_c(n)$  and the decision symbol  $d(n)$  is calculated and can be expressed with  $p(n)$ ;

$$p(n) = \text{Im} \left[ \frac{r_c(n)}{d(n)} \right]$$

$$\cong \sin \left( \theta_f(n) + \theta_p(n) + \theta_v(n) - \theta_d(n) \right) \quad (4)$$

where  $\theta_f(n)$  is phase difference due to the frequency offset,  $\theta_p(n)$  is caused by phase offset,  $\theta_v(n)$  is the phase noise due to  $v(n)$ , and  $\theta_d(n)$  is the phase of the decision symbol  $d(n)$ . When the carrier frequency offset  $\Delta f > 0$ , the phase difference between  $p(n)$  and the previous phase  $p(n-1)$  is expected to be positive, or vice versa. Hence, the phase difference between two successive phase detector outputs is related to the frequency error. Mathematically, the phase difference between  $p(n)$  and  $p(n-1)$  is proportional to the frequency offset error, which is given by,

$$p(n) - p(n-1) = \Delta f + \theta_v(n) - \theta_v(n-1) \quad (5)$$

where  $\theta_v(n)$  and  $\theta_v(n-1)$  are zero-mean noise components. Therefore, the tentative phase difference value is used to provide an approximate steady-state condition. Yet, the phase will keep changing because of the residual frequency offset. Due to this change, during crossing the decision boundaries, wrong decision symbol estimations may be chosen. When the varying phase offset  $p(n)$  crosses the decision boundary, the sign of the phase will change and the successive difference between these two phase offset values will cause wrong decisions. To eliminate this problem, stop-and-go method (S&G) is used. This algorithm is very powerful for compensating the wrong estimations during the transition in the decision boundaries [5]. The output of the S&G algorithm changes with the signs of two successive phase offset estimations. If the signs are opposite then the previous calculated phase offset value is used again.

$$p_e(n) = \begin{cases} C \cdot (p(n) - p(n-1)) & \text{sgn}(p(n)) = \text{sgn}(p(n-1)) \\ p_e(n-1) & \text{sgn}(p(n)) \neq \text{sgn}(p(n-1)) \end{cases} \quad (6)$$

where  $\text{sgn}(x)$  represents the signum function for variable  $x$  and  $C$  is a positive constant that impacts the acquirement duration.

Finally, a differentiator is used to detect the mean value of increment or reduction in phase offset. The tentative frequency offset error is used to correct the signal for each step until the steady-state is achieved. In conclusion, the carrier frequency offset detector outputs two phase offsets; one is the phase offset value that is used to correct every successive symbol to be able to track the residual frequency offset value, and the other is the input of the differentiator which is used for compensating the effect of the frequency offset error.

## 4.2. Rectangular Constellation Diagrams

For the rectangular mappings, a similar kind of approach is used. The method used is comparable with [4]. The first step of the method contains an energy detector to choose the symbols having the highest SNR which lies on the corners of the constellation mapping, since they have the highest energy. Then, the symbols having the highest SNR value is passed through a polarity detector to find the phase rotation due to both frequency and phase offsets. A close form of expression for polarity detector is given in (4). The value of  $p(n)$  indicates the rotation of the constellation lying on the outermost corners, i.e. polars. The frequency offset on this symbol is related to the symbol index and is given by,

$$f_o(n) = \frac{p(n)}{2\pi n T_s} \quad (7)$$

where  $T_s$  is the symbol period. Then,  $f_o(n)$  is used to remove the frequency offset of the next symbol,  $r(n+1)$ . The general expression for the frequency offset correction is,

$$r(n+1) = r(n+1)e^{-j2\pi n f_o(n) T_s} \quad (8)$$

where  $r(n)$  denotes the received symbol sequence.

32-QAM differs from 16-QAM and 64-QAM in the mapping type. 16-QAM and 64-QAM both have square constellation mappings where as 32-QAM has a constellation mapping similar to the shape of a plus sign and can be referred as non-square mapping. There are different algorithms for square and non-square mappings, but the procedure is the same. Outermost constellations are chosen in non-square mappings as well. Instead of finding the phase rotation with respect to the polars, the phase rotation is calculated by looking at the phases of the constellation points in the outermost constellation points. There are four useful constellations in a square mapping, where as in a non-square mapping there are eight useful constellations.

When the modulation order increases, performance of the algorithm degrades since the probability of getting a constellation having the highest SNR will decrease. To prevent this problem some methods are proposed, though they are proven to be non-efficient.

## 5. SIMULATION RESULTS

The simulations are performed with various QAM types in an AWGN communication channel. The roll-off factor of the pulse shaping raised cosine filter,  $\alpha$ , is 0.35 and the oversampling rate is 16. The results are evaluated between 0 dB and 30 dB SNR and the quality of the frequency offset estimations are measured by the normalized root mean square error (NRMSE). NRMSE is defined as;

$$NRMSE = \sqrt{E[(1 - f'_o/f_o)^2]} \quad (9)$$

where  $f'_o$  represents the estimated frequency offset value and  $f_o$  is the exact offset value. The frequency offset is chosen to be 1/1000 of the symbol rate. Throughout simulations, it is seen that the recovery range can go up to 1/100 of the symbol rate for rectangular mappings.

The modulation identification process depends on the number of the rings generated by the frequency offset and their densities as indicated. The number of rings is determined by the number of clusters generated by the k-means algorithm. Some trials are performed with two different cluster numbers, eleven and fifteen. Throughout these trials, it is seen that, with eleven clusters false alarm rates are higher. For this reason, fifteen clusters locations will be generated for grouping purpose.

Table 1: Modulation Type vs. False Alarm Rate

| Modulation Type    | False Alarm Rate |
|--------------------|------------------|
| Rectangular-16-QAM | 0%               |
| Circular-16-QAM    | 0%               |
| v29-16-QAM         | 0%               |
| Star-16-QAM        | 0%               |
| Rectangular-32-QAM | 0%               |
| Circular-32-QAM    | 0%               |
| Rectangular-64-QAM | 1%               |
| Circular-64-QAM    | 3%               |

Fifteen clusters are generated by this function and due to the nature of this function, i.e. lack of generating exact clusters, close clusters are grouped together to form a single cluster. Then, for specific modulation types such as circular-16-QAM and star-16-QAM, at each cluster location, the density information of the amplitude of received signal is extracted and compared to each other. Using these information, the modulation order and constellation mapping is determined and fed to the frequency estimation block. The false alarm table for fifteen clusters is given in Table 1. The false alarm rates are calculated at 19dB for 16-QAM, 25dB for 32-QAM and 30dB for 64-QAM mappings. The simulations for different modulation orders are performed with different SNR values because in order to demodulate them perfectly these are the required values.

After the modulation order estimation process, the frequency offset is corrected. By using the knowledge of modulation order, two different frequency offset correction algorithms are applied for different orders.

The performance of the first algorithm, which is applied to circular mappings, depends on the number of constellation points in the outermost ring. If the number of decision symbols increases, number of decision boundaries also increases which causes degradation in the system performance. If the outermost ring has more constellation

points, decision symbols and decision boundaries become closer to each other. Consequently, trying to reach the steady state becomes harder for low SNR values when the number of constellation points in the outermost ring increases. As can be seen clearly from Fig. 3, v29-16-QAM and star-16-QAM achieves steady state with less SNR than circular-16-QAM. However, having more number of points in the outermost ring will increase the probability of having a symbol exceeding the threshold value, which enables to track the frequency error easier.

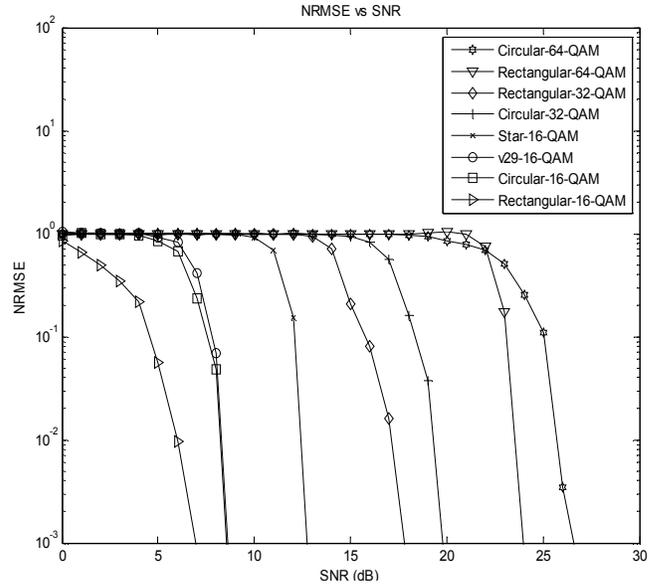


Fig. 3: NRMSE Values for the Estimated Frequency Offset for Different SNR Values

Moreover, the second method, which uses polarity detector, does not depend on the outermost constellation point number, since it uses constant decision symbols. However, reaching the steady state becomes harder for higher orders due to decreasing the probability of getting a symbol that exceeds the threshold value. As the performance difference between rectangular-16-QAM, rectangular-32-QAM and, rectangular-64-QAM in Figure 3, shows the effect.

## 6. CONCLUSION

In this study, modulation identification as well as frequency and phase offset problems are treated jointly for QAM signals. The distribution of amplitude and density information for each distribution is a key property for QAM signals since each of the mappings has different properties. Using this property of QAM signals, first, the modulation order is determined while there is frequency and phase offsets are present in the system, then using a suitable frequency offset algorithm, the frequency offset and the

phase offset are removed from the system. By skillfully introducing every step, one of the most challenging and common problems for blind receivers has been interpreted and solved.

## 7. REFERENCES

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