

# ADAPTIVE ROLL-OFF FACTOR UTILIZATION FOR FMT-BASED FBMC BURST STRUCTURES

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**Abstract**—In this paper, we propose an adaptive roll-off factor utilization for filtered multitone (FMT) based filter bank multicarrier (FBMC) burst structures. Conventionally, a single prototype filter which has the same roll-off factor is employed for whole FBMC symbols. Thus, the conventional approach neglects the advantage of using filter adaptation against doubly dispersive channels. Unlike the conventional approach, in this study, different filters in terms of roll-off factors are utilized within the burst and the roll-off factors are adaptively changed by paying regard to the time and frequency dispersions of the channels. Also, by allowing the controlled interference between subcarriers, an average frequency spacing is applied to the burst structure. Therefore, immunity against multipath delay spread or Doppler spread is gained. Additionally, new degree of freedoms, i.e., average frequency spacing, adaptation speed, and filter truncation are investigated for the FMT-based burst structure.

**Index Terms**—Burst structure, FBMC, FMT, roll-off factor, root raised cosine, truncation.

## I. INTRODUCTION

Consistently increasing number of users along with wide variety of wireless communication applications, and extensive growth on the user demands from the wireless communication systems lead more adaptive, flexible, and efficient future radio access techniques. Especially, over the last two decades, radio access techniques which are able to exploit the multidimensional electrospace, e.g. orthogonal frequency division multiplexing (OFDM), have been heavily investigated in the literature. Recently, filter bank multicarrier (FBMC) technique which was introduced by Chang and Saltzberg in 1960s [1], [2], is re-considered as a tempting solution for future radio access technologies because of its flexibility on pulse shapes along with filters well-localized in frequency. Basically, the flexibility of choosing any prototype filter to combat inter-symbol interference (ISI) and inter-carrier interference (ICI) in doubly dispersive channels makes FBMC an attractive choice over OFDM.

It is possible to generate FBMC symbols with different modes, i.e., filtered multitone (FMT), staggered multitone (SMT), and cosine-modulated multitone (CMT) [3]. FMT-based FBMC is a multicarrier scheme that has been proposed before for DSL applications [4], [5]. In this mode, subcarriers are separated with guard bands instead of overlapping of adjacent subcarriers as in SMT and CMT modes. Therefore, FMT-based FBMC does not provide the same efficiency

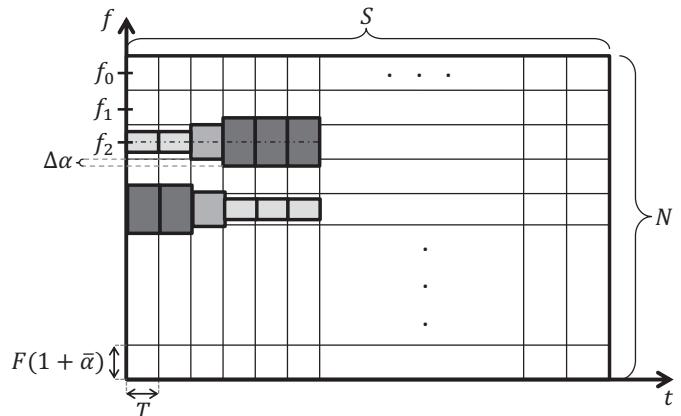


Fig. 1: Proposed burst structure.

of SMT and CMT modes because of guard bands. However, FMT brings flexibility on multiple-input multiple-output (MIMO) channels which makes an attractive scheme for next generation communications systems.

In FBMC, time and frequency characteristics of transmitting and receiving filters are the critically important for the performance of the systems. For example, consider an FBMC symbol constructed with root-raised cosine (RRC) filter. Basically, roll-off factor ( $\alpha$ ) determines the time and frequency characteristics of RRC filter. If small  $\alpha$  is applied to the filter, higher sidelobes are obtained in time domain and makes the constructed FMT-based FBMC symbols more susceptible to time varying channels.

In the literature, existing methods are based on designing prototype filter to achieve better sidelobe suppression for perfect reconstruction [6], [7]. In [8], the effects of roll-off factor and truncation are also analyzed. These methods are useful when only a single prototype function is applied for whole burst structure. However, use of fixed  $\alpha$  factors ignores the adaptations on time-frequency varying channels. In this study, instead of using single prototype filter for the whole burst,  $\alpha$  value is changed by  $\pm\Delta\alpha$  for the consecutive FBMC symbol by paying regard to the orthogonality loss due the change on  $\alpha$  factor as given in Fig. 1. Therefore, desired  $\alpha$  on each subcarrier is obtained by changing  $\alpha$  factor gradually within the range between  $\alpha_{min}$  and  $\alpha_{max}$  specified according to transmission quality requirements. In addition

to  $\alpha$  adaptation, interference between subcarriers is allowed in the proposed burst structure. Frequency spacing, which is directly related with ICI, is determined according to an average  $\bar{\alpha}$  factor as  $F(1 + \bar{\alpha})$ . The main contributions of proposed method are given as follows;

- 1) Different  $\alpha$  factors are applied to the burst structure. Therefore, instead of using single prototype function, advantage of controlling the filter in time and frequency domains is utilized in the burst structure.
- 2) By allowing controlled interference, new degree of freedoms, i.e.,  $\bar{\alpha}$ ,  $\Delta\alpha$ , and truncation are introduced and investigated.

This paper is organized as follows: FMT-based FBMC system modeling is presented in Section II. In Section III, orthogonality of the burst structure is analyzed considering different parameters, i.e.,  $\bar{\alpha}$ ,  $\Delta\alpha$ , and filter length. Then, corresponding burst designing procedure is explained. Finally, simulation results are given by using the proposed burst structure.

## II. SYSTEM MODEL

Analytical expression of the FMT-based baseband transmitted signal can be given by

$$x(t) = \sum_{m=-\infty}^{\infty} \sum_{k=0}^{N-1} X_{mk} p(t - mT) e^{j2\pi k F t}, \quad (1)$$

where  $N$  is the number of subcarriers,  $X_{mk}$  is the complex symbol located on the subcarrier indicated by the time index  $m$ , and the frequency index  $k$ ,  $T$  is the symbol spacing,  $F$  is the subcarrier spacing, and  $p(t)$  is the RRC prototype filter as

$$p(t) = \begin{cases} 1 - \alpha_{mk} + \frac{4\alpha}{\pi}, & t = 0 \\ \frac{\alpha}{\sqrt{2}} \left[ \left(1 + \frac{2}{\pi}\right) \sin\left(\frac{\pi}{4\alpha}\right) + \left(1 - \frac{2}{\pi}\right) \cos\left(\frac{\pi}{4\alpha}\right) \right], & t = \pm \frac{T}{4\alpha} \\ \frac{\sin\left((1-\alpha)\frac{\pi t}{T}\right) + \frac{4\alpha t}{T} \cos\left((1+\alpha)\frac{\pi t}{T}\right)}{\frac{\pi t}{T} \left(1 - \frac{16\alpha^2 t^2}{T^2}\right)}, & |t| \leq T_f \\ 0 & \text{otherwise} \end{cases}, \quad (2)$$

where  $T_f$  is filter duration as  $T_f = KT$  and  $K$  is the filter length as a multiple of symbol spacing. In FMT mode, in order satisfy the orthogonality,  $F$  has to be selected at least as  $F = \frac{1}{T}(1 + \bar{\alpha})$  if  $K = \infty$ .

After the signal passes through the linear time-varying wireless channel  $h(t)$ , the received signal is obtained as

$$y(t) = \int_{-\infty}^{\infty} h(t)x(t - \tau)d\tau. \quad (3)$$

Received symbol  $\tilde{X}_{nl}$  located on time index  $n$  and frequency index  $l$  is obtained by the projection of the received signal on analysis function  $\hat{g}_{nl}(t)$  as

$$\tilde{X}_{nl} = \langle y(t), \hat{g}_{nl}(t) \rangle, \quad (4)$$

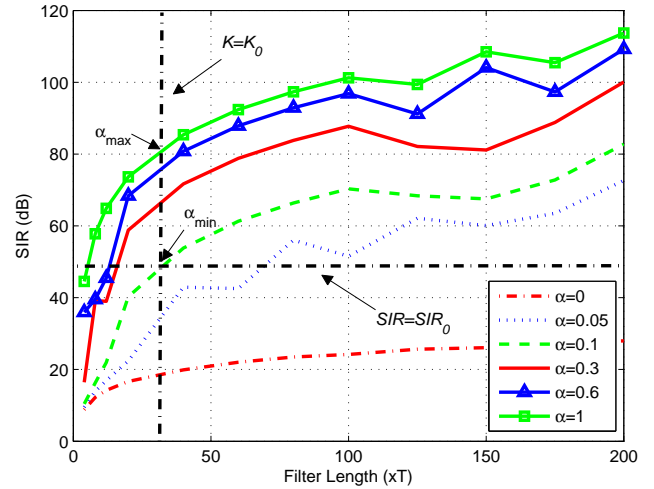


Fig. 2: The impact of truncation on SIR for different  $\alpha$ . Filter length is identified with respect to the system constraints, i.e. maximum  $K$  and minimum desired SIR.

where

$$\hat{g}_{nl}(t) = p(t - nT) e^{j2\pi l F t}. \quad (5)$$

In (4),  $\tilde{X}_{nl}$  indicates the estimated symbol.

## III. DESIGNING BURST STRUCTURE

Theoretically, convolution of two RRC filters with infinite time responses satisfies Nyquist criterion. However, infinite time response cannot be satisfied because of the implementation issues. Thus, employing truncated RRC filter causes orthogonality loss which results in ISI and ICI. As given in Fig. 2, signal-to-interference ratio (SIR) is strongly depended on  $K$ . Therefore, precautions that can compensate the impairment due to truncation must be considered.

Another issue resulting in orthogonality loss within the proposed frame structure is to employ different  $\alpha_{mk}$ . In the conventional approaches, only a single prototype filter is used for all symbols at the transmitter and receiver. Therefore, the information symbols are decoded at receiver based on Nyquist criterion. However, since the proposed frame structure introduces an adaptive structure with  $\alpha_{mk}$  for each  $m$  and  $k$ , orthogonality is lost between adjacent symbols in time domain for FMT-based FBMC systems. In addition to that, even if the transmitter and the receiver filters are matched, the tails of adjacent symbols that have different  $\alpha_{mk}$  values spoil the perfect reconstruction. Thus, in order to limit the interference due to the tails of different  $\alpha_{mk}$ , the variation of the  $\alpha_{mk}$  factor for the consecutive symbols is limited and gradually changed with  $\mp\Delta\alpha$ . As shown in Fig. 3, the smoother transitions in  $\alpha_{mk}$  also result in smoother SIR changes. On the contrary, the sharp transitions in  $\alpha_{mk}$  cause more interference and larger variation of SIR.

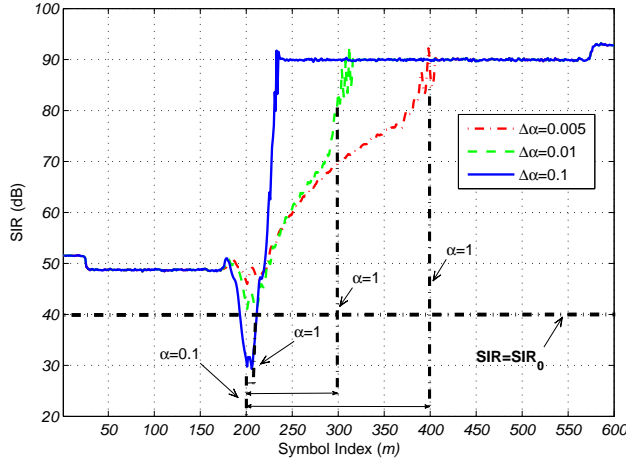


Fig. 3: The impact of  $\Delta\alpha$  on instantaneous SIR in the burst structure ( $K = 40$ ). Faster changes in  $\alpha$ , i.e., greater  $\Delta\alpha$ , causes high SIR degradation but quick SIR compensation in time.

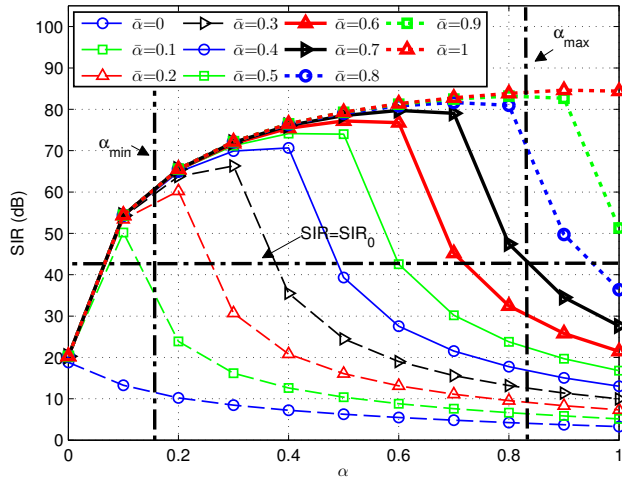


Fig. 4: The impact of  $\bar{\alpha}$  on SIR for different  $\alpha$ . After identifying  $\alpha$  range using Fig. 2, the required subcarrier spacing, i.e.,  $(1+\bar{\alpha})$  is obtained by considering  $SIR_0$  and  $f_\alpha(\alpha)$ .

Under these circumstances, three main parameters,  $K$ ,  $\bar{\alpha}$  and  $\Delta\alpha$ , are critically important to maintain SIR above a minimum value,  $SIR \geq SIR_{min}$ . In addition,  $\alpha_{min}$  and  $\alpha_{max}$  must be specified by considering the orthogonality loss. First, in order to see the effect of  $K$ , SIR curves are given in Fig. 2 for different  $\alpha_{mk}$  and  $K$ . Fig. 2 provides a selection space to specify  $\alpha_{min}$  by showing the limitations of system with two lines pointing to the maximum filter length that can be generated by the hardware and minimum desired

SIR which are indicated by  $K_0$  and  $SIR_0$ . Then,  $\Delta\alpha$  is determined as the second main parameter. The impact of  $\Delta\alpha$  on SIR is obtained in Fig. 3 using the greatest transition in terms of  $\alpha_{mk}$  variation from predetermined  $\alpha_{min}$  to  $\alpha_{max}$  for different  $\Delta\alpha$ . For example,  $\alpha_{min}$  and  $\alpha_{max}$  are set to 0.1 and 1 in Fig. 3, respectively. Note that faster changes in  $\alpha$  causes high SIR degradation but quick SIR compensation in time.

The final step to complete the proposed design structure is to determine  $\bar{\alpha}$ . In Fig. 4, SIR is calculated for different  $\alpha_{mk}$  for given  $\bar{\alpha}$ . Note, each curve is constructed for bursts whose all symbols have corresponding  $\alpha_{mk}$  value. However, in real implementations,  $\alpha_{mk}$  takes random values between  $\alpha_{min}$ - $\alpha_{max}$ . Therefore, if  $\alpha_{max}$  and the probability density function (PDF) of  $\alpha_{mk}$  ( $f_\alpha(\alpha)$ ) in the system are known, minimum required  $\bar{\alpha}$  can be obtained by calculating the expected SIR value of each curve ( $\overline{SIR}(\bar{\alpha})$ ) in Fig. 4 as

$$\bar{\alpha}_{min} = \arg \min_{\bar{\alpha}} (|\overline{SIR}(\bar{\alpha}) - SIR_0|) \quad (6)$$

where

$$\overline{SIR}(\bar{\alpha}) = 10 \log \left( \frac{1}{\left( \int_{\alpha_{min}}^{\alpha_{max}} \alpha \sqrt{10^{-\frac{SIR_{\bar{\alpha}}(\alpha)}{10}}} f_\alpha(\alpha) d\alpha \right)^2} \right) \quad (7)$$

In (7),  $\overline{SIR}(\bar{\alpha})$  and  $SIR_{\bar{\alpha}}(\alpha)$  indicate the average SIR for a given  $f_\alpha(\alpha)$ , SIR for given  $\alpha$  and  $\bar{\alpha}$  as in Fig. 4, respectively.

When all of the required parameters are determined properly, the adaptive burst structure satisfying  $SIR_0$  requirement with minimum bandwidth can be designed.

#### IV. SIMULATIONS

A design example is given in order to verify described procedure with computer simulations and to investigate the performance of the proposed burst structure. The burst is designed with 330 FBMC symbols and 16 subcarriers. Roll off factor of each subcarrier's symbols' vary for 3 times. The transmission quality requirement, i.e.,  $SIR_0$ , is selected as 40 dB. If the maximum filter length  $K_0$  is assumed as 40 because of the hardware limitations, as shown in Fig. 2,  $\alpha_{min}$  needs to be larger than 0.1 and  $\alpha_{max}$  can be selected as 1 without any constraint. On the other hand,  $\Delta\alpha$  is determined 0.01 to keep the SIR more than 40 dB using Fig. 3. If PDF of  $\alpha_{mk}$  is assumed as uniform heuristically,  $\bar{\alpha}$  is calculated as 0.65 using (6). When the burst is designed with these parameters, average SIR is obtained as 47.8 dB which satisfies desired transmission quality. For different  $SIR_0$  values, a list of determined parameters and measured SIR values are given in TABLE I ( $\alpha_{max} = 1$ ,  $\Delta\alpha = 0.01$ ).

#### V. CONCLUDING REMARKS

In this paper, an adaptive FMT based FBMC burst structure is proposed to compensate the impairments of doubly

$SIR_0$ (Theoretical)	$\alpha_{min}$	$\bar{\alpha}$	SIR (Simulation)
20 dB	0.05	0.35	23.4 dB
30 dB	0.05	0.55	39.8 dB
50 dB	0.2	0.75	57.4 dB

TABLE I: Simulation results for different  $SIR_0$  values ( $\alpha_{max} = 1$ ,  $\Delta\alpha = 0.01$ ).

dispersive channels to maintain SIR. Roll off factors of the symbols in the proposed burst are changing adaptively within an  $\alpha$  range specified according to transmission requirement. Also, subcarriers are allowed to be overlapped considering for a given SIR level. By employing the proposed burst structure, the spectral efficiency of the burst is increased by adjusting the subcarrier spacing according to the PDF of the roll-off in the system, instead of adjusting for the worst case. By introducing  $\alpha$  adaptation and partially overlapping between subcarriers by maintaining SIR, new degree of freedoms in the burst structure are achieved. Especially, if the introduced degree of freedoms can be exploited with scheduling algorithms and adaptive modulation to yield better proposed structure. As an extension of this paper, we will discuss the statistics of roll-off considering the channels of the users and the impact of scheduling on proposed burst structure.

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