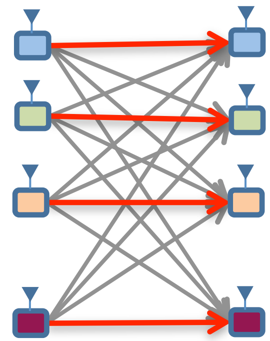
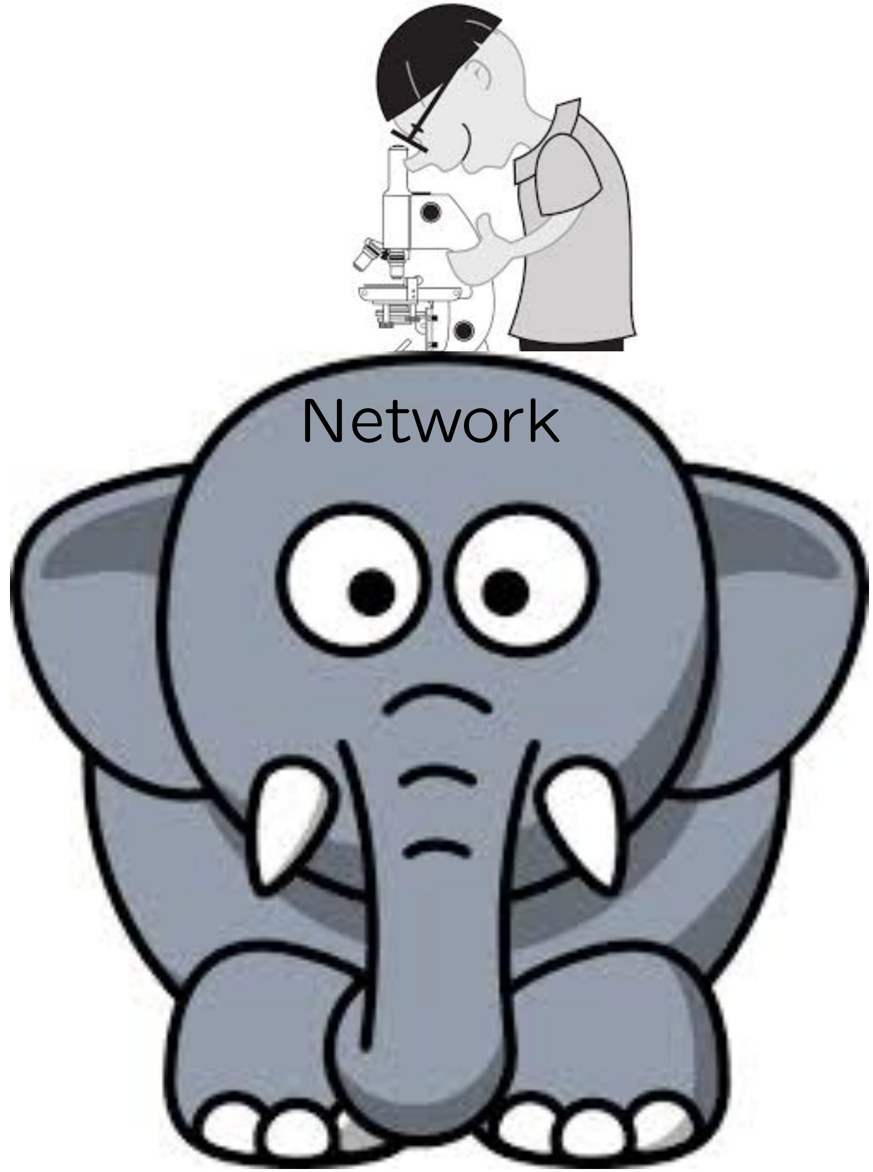
Robust Interference Management – An Information Theoretic Perspective



Syed A. Jafar
CPCC, EECS, UC Irvine

International Workshop on Emerging Technologies for 5G Wireless Cellular Networks

Exact Capacity



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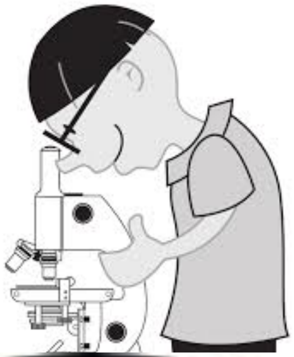
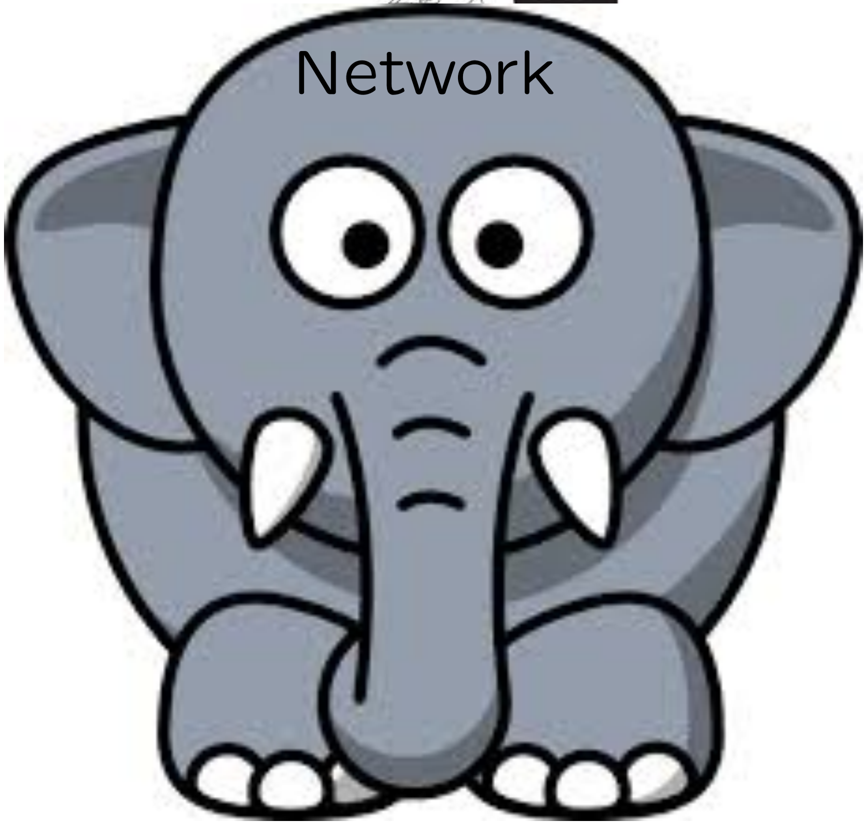




Approximate Capacity

Degrees of Freedom (DoF)

Degrees of Freedom (DoF).

Generalized notion that encompasses Bandwidth, Spectrum, Multiplexing Gain, Capacity Pre-log, Number of Signal Dimensions.

$$\text{DoF} = \lim_{\text{SNR}\to\infty} \frac{\text{Network Capacity}}{\text{Link Capacity}}$$

$\approx$ No. of interference free links that can be created in a network.

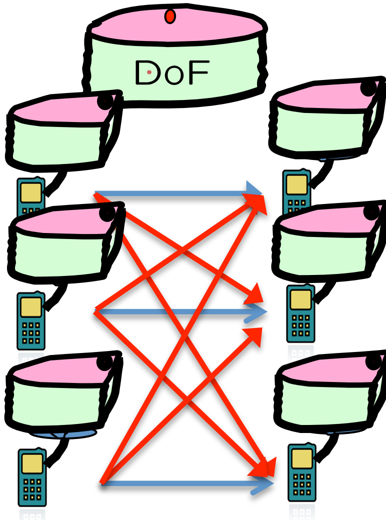
CSIT. Channel State Information at the Transmitter(s).

CSIR. Channel State Information at the Receiver(s).

Focus on CSIT.
Assume perfect CSIR.

How do the DoF scale per user?

Perfect CSIT



Host-Madsen-Nosratinia Conjecture (2005): DoF Collapse.

Collapse means DoF $= 1$.
Optimal to serve one user at a time (e.g., TDMA).

Disproved, DoF $= K/2$ (Everyone gets half the cake).

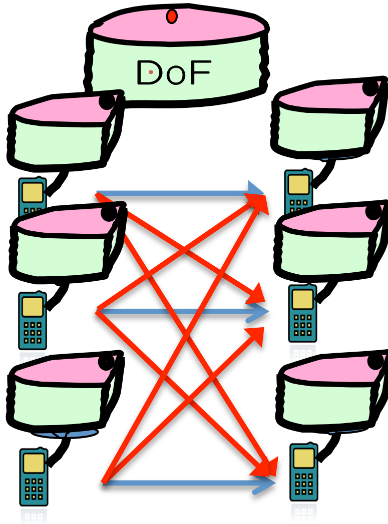
V. Cadambe, S. Jafar, Interference Alignment and the Degrees of Freedom of the K user Interference Channel, IEEE Trans. on Info. Theory, 54(8):3425-3441, Aug. 2008.

Interference Alignment ✔

How do the DoF scale per user?

Compound CSIT

(channel uncertainty among finitely many possibilities)



Weingarten-Shamai-Kramer Conjecture (2007):

K user MISO BC with Finite State Compound CSIT DoF $= 1$

DoF collapse even with full cooperation/data-sharing.

Disproved, DoF $= \frac{K^2}{(2K-1)}$ (still approx. half the cake per user)

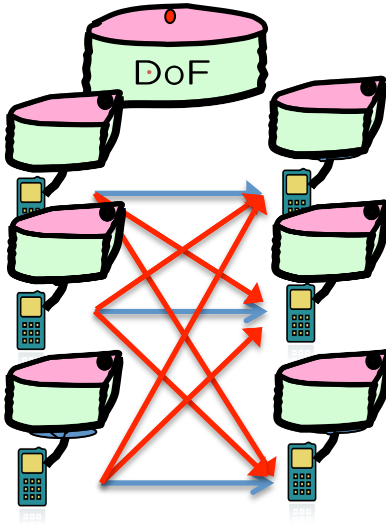
T. Gou, S. Jafar, C. Wang, On the Degrees of Freedom of Finite State Compound Wireless Networks, IEEE Trans. on Info. Theory, 57(6): 3286-3308, June 2011.

M. Maddah-Ali, On the degrees of freedom of the compound MISO broadcast channels with finite states, IEEE Int. Symp. on Info. Theory ISIT 2010, 2273-2277.

Zero Forcing ✘
Data Sharing ✘
Interference Alignment ✔

How do the DoF scale per user?

Finite Precision CSIT



Lapidoth-Shamai-Wigger Conjecture (2005): DoF Collapse.

Collapse even with full cooperation among transmitters.

(MISO BC)

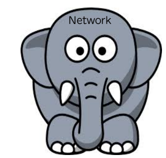
ITA 2006 Open Problem

Proved

A. Gholamidavoodi, S. Jafar, Aligned Image Sets under Channel Uncertainty: Settling a Conjecture by Lapidoth, Shamai and Wigger on the Collapse of Degrees of Freedom under Finite Precision CSIT, ArXiv:1403.1541.

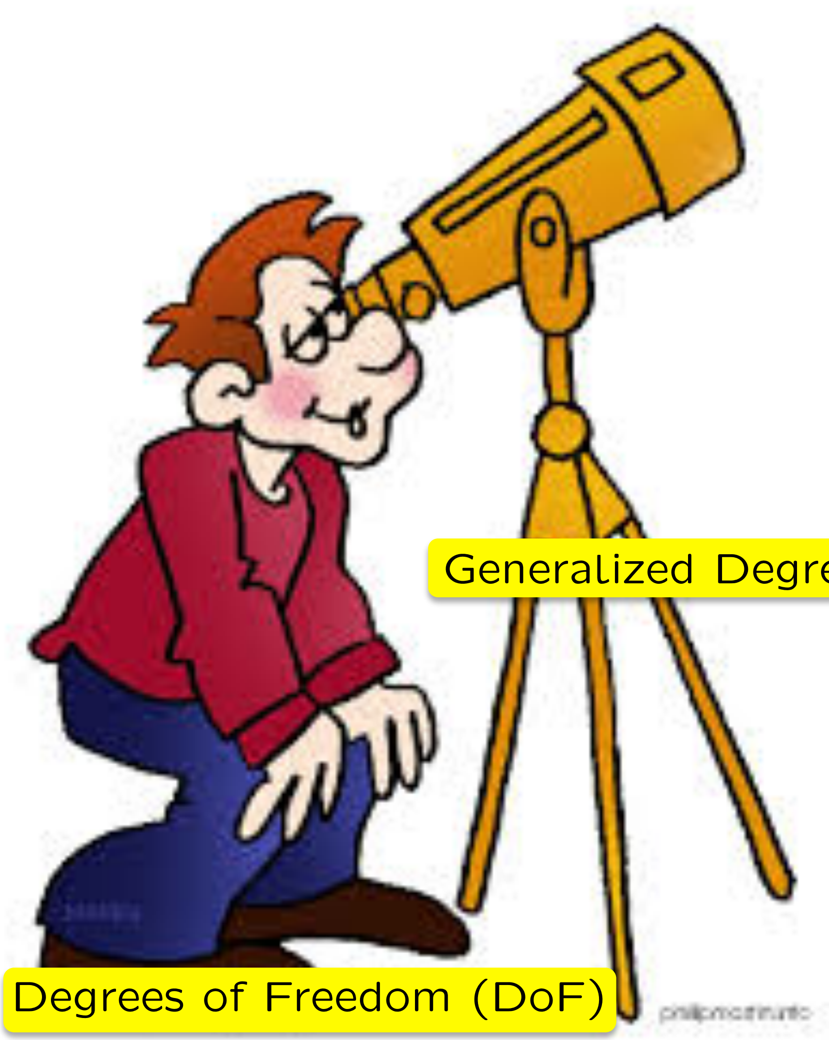
☺ Perfect CSIT: Everyone Gets Half the Cake!

☹ Finite Precision CSIT: DoF Collapse!



Capacity

Bounded Gap Approximations



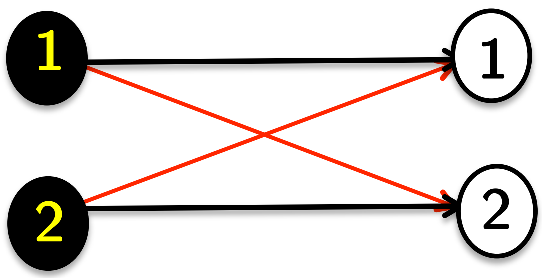
Generalized Degrees of Freedom (GDoF)

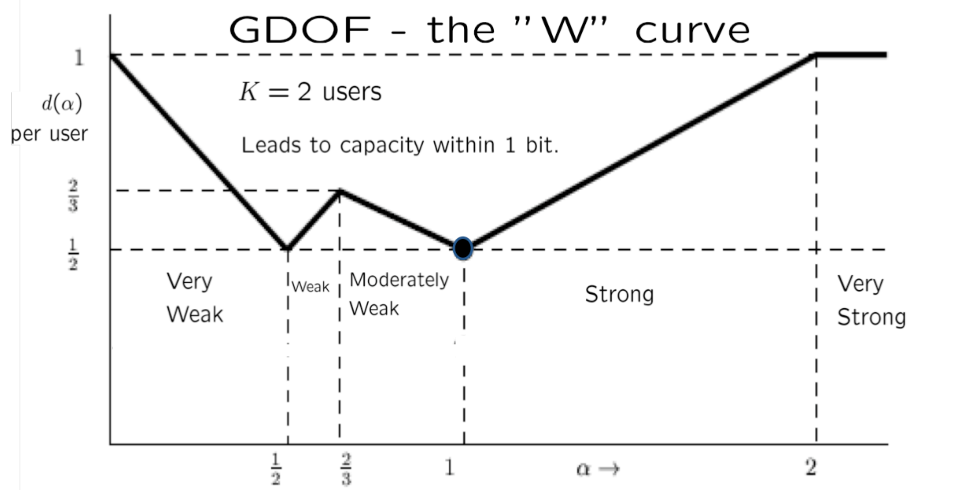
Degrees of Freedom (DoF)

Progressive Refinements Approach

DoF versus GDoF

Two User Interference Channel





DoF

$$
\begin{aligned}
Y_1 &= \sqrt{P}h_{11}X_1 + \sqrt{P}h_{12}X_2 + Z_1 \\
Y_2 &= \sqrt{P}h_{21}X_1 + \sqrt{P}h_{22}X_2 + Z_2
\end{aligned}
$$

DoF (fixed channel, $P \rightarrow \infty$)

DoF $= 1$.
(one user at a time)

What about weak interference?

What about strong interference?

Implicitly assumes
all signals equally strong.

GDoF

$$
\begin{aligned}
Y_1 &= \sqrt{P^{\alpha_{11}}}h_{11}X_1 + \sqrt{P^{\alpha_{12}}}h_{12}X_2 + Z_1 \\
Y_2 &= \sqrt{P^{\alpha_{21}}}h_{21}X_1 + \sqrt{P^{\alpha_{22}}}h_{22}X_2 + Z_2
\end{aligned}
$$

GDoF (channel changes, $P \rightarrow \infty$)

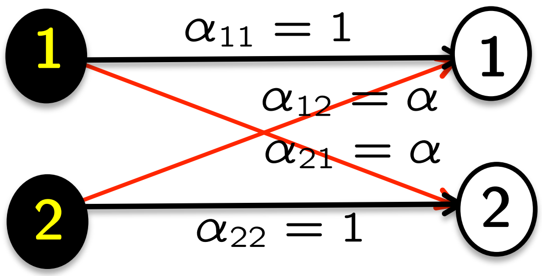
Covers all (weak/strong) interference regimes.

The GDoF model leads to capacity within 1 bit.

R. Etkin, D. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," IT Trans.2008.54(12):5534–5562.

Leads to exact capacity in noisy regime.

Finite Precision CSIT: DoF Collapse.
What about **GDoF under Finite Precision CSIT**?



X-channel: Interference Alignment

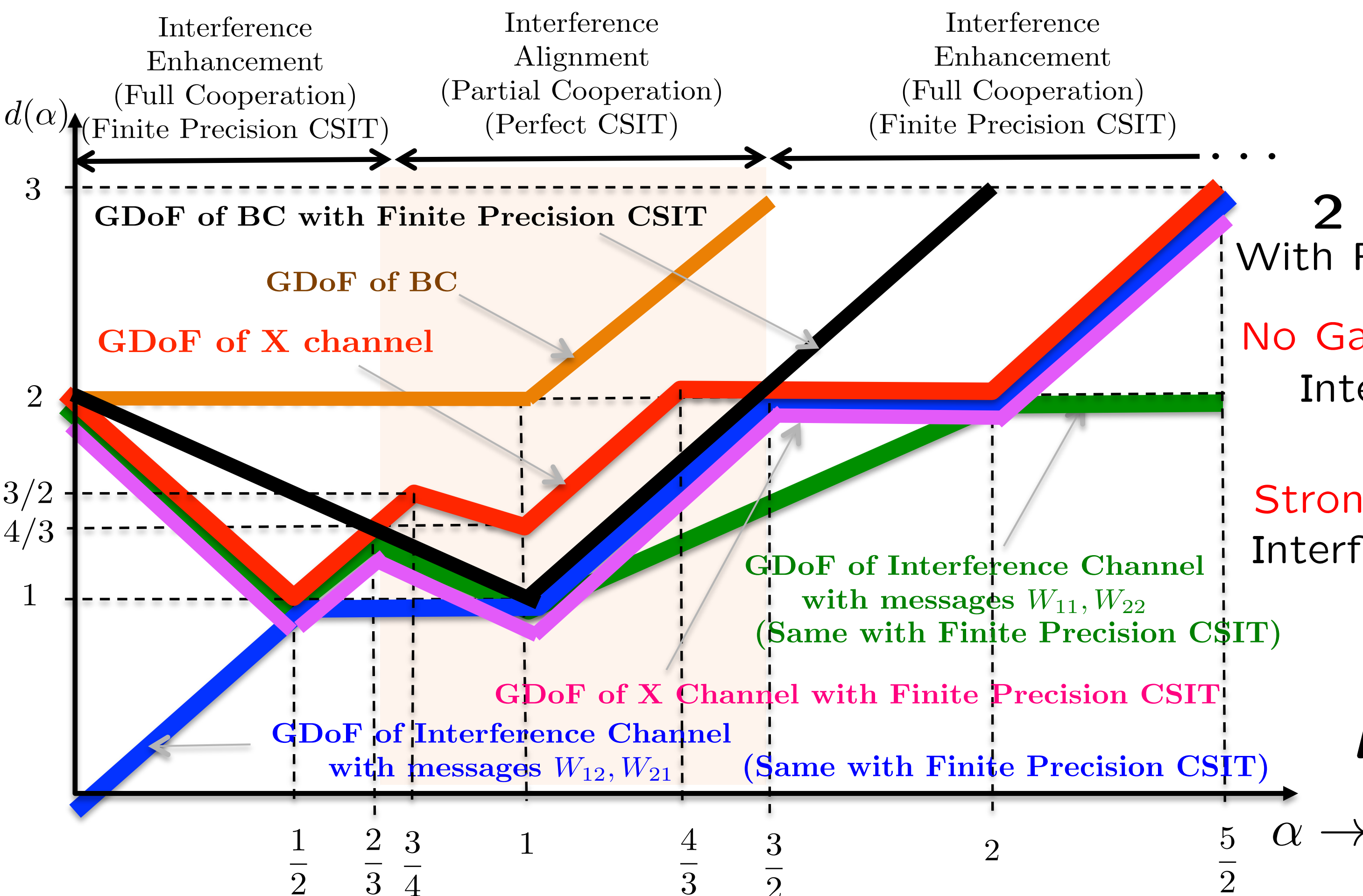
MISO BC: Zero Forcing

[Gholamidavoodi, Jafar, GC'15]

GDoF:

$$Y_1 = \sqrt{P^{\alpha_{11}}} h_{11} X_1 + \sqrt{P^{\alpha_{12}}} h_{12} X_2 + Z_1$$
$$Y_2 = \sqrt{P^{\alpha_{21}}} h_{21} X_1 + \sqrt{P^{\alpha_{22}}} h_{22} X_2 + Z_2$$

h_{ij} only known to finite precision.



2 User Setting
With Finite Precision CSIT

No Gains From:
Interference Alignment
Zero Forcing

Strong Gains From:
Interference Enhancement
Data-sharing

K Users?

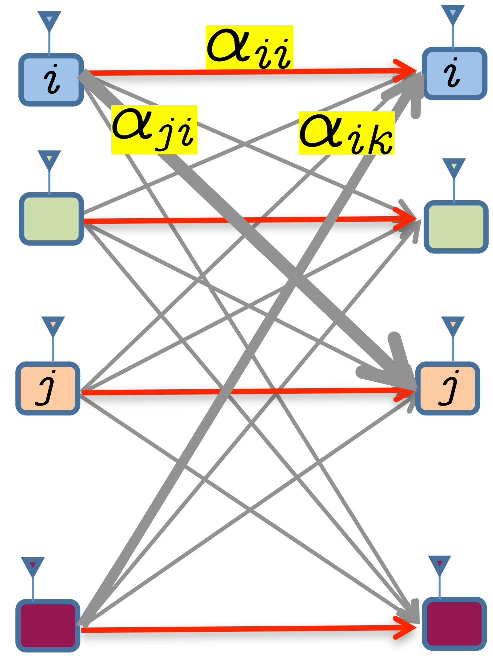
Finite Precision CSIT: DoF Collapse.

What about **GDoF under Finite Precision CSIT**?

K Users?

(Open Problem)

"Curse of Dimensionality"



$$Y_k = \sqrt{P^{\alpha_{kk}}} h_{kk} X_k + \sum_{i \in [K]/k} \sqrt{P^{\alpha_{ki}}} h_{ki} X_i + Z_k$$

Strong Interference

S. Jafar, Topological Interference Management through Index Coding, IEEE Trans. on Information Theory, 60(1):529-568, Jan. 2014.

Weak Interference

C. Geng, N. Naderializadeh, S. Avestimehr, S. Jafar, On the Optimality of Treating Interference as Noise, IEEE Trans. on Information Theory, 61(4):1753-1767, April 2015.

Practical Principles of Interference Management

Ignore

interference that is sufficiently weak

↓

Treat Interference as Noise (TIN)

↓

Optimal?

Avoid

interference that is not

↓

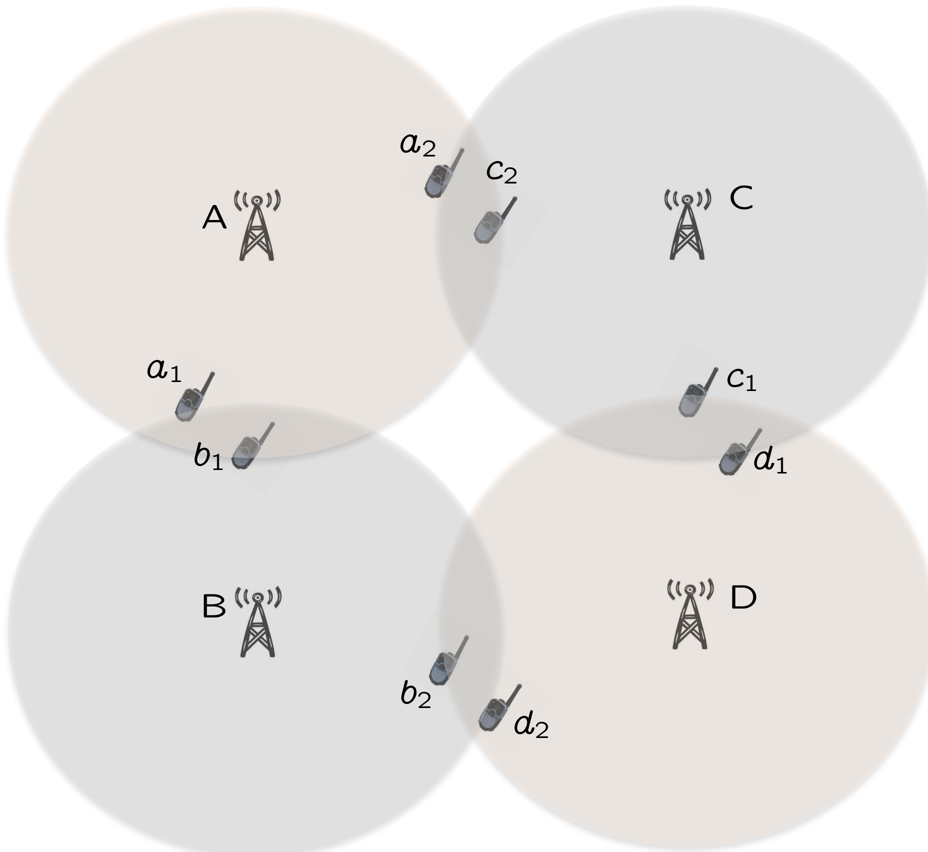
TDMA/FDMA/CDMA

↓

Optimal?

Optimal Interference Avoidance

Topological Interference Management



A base station can be heard only within own cell.

Cell edge users located near the boundaries.

Transmitters (BS) know only connectivity.

No knowledge of channel coefficient values for connected users.

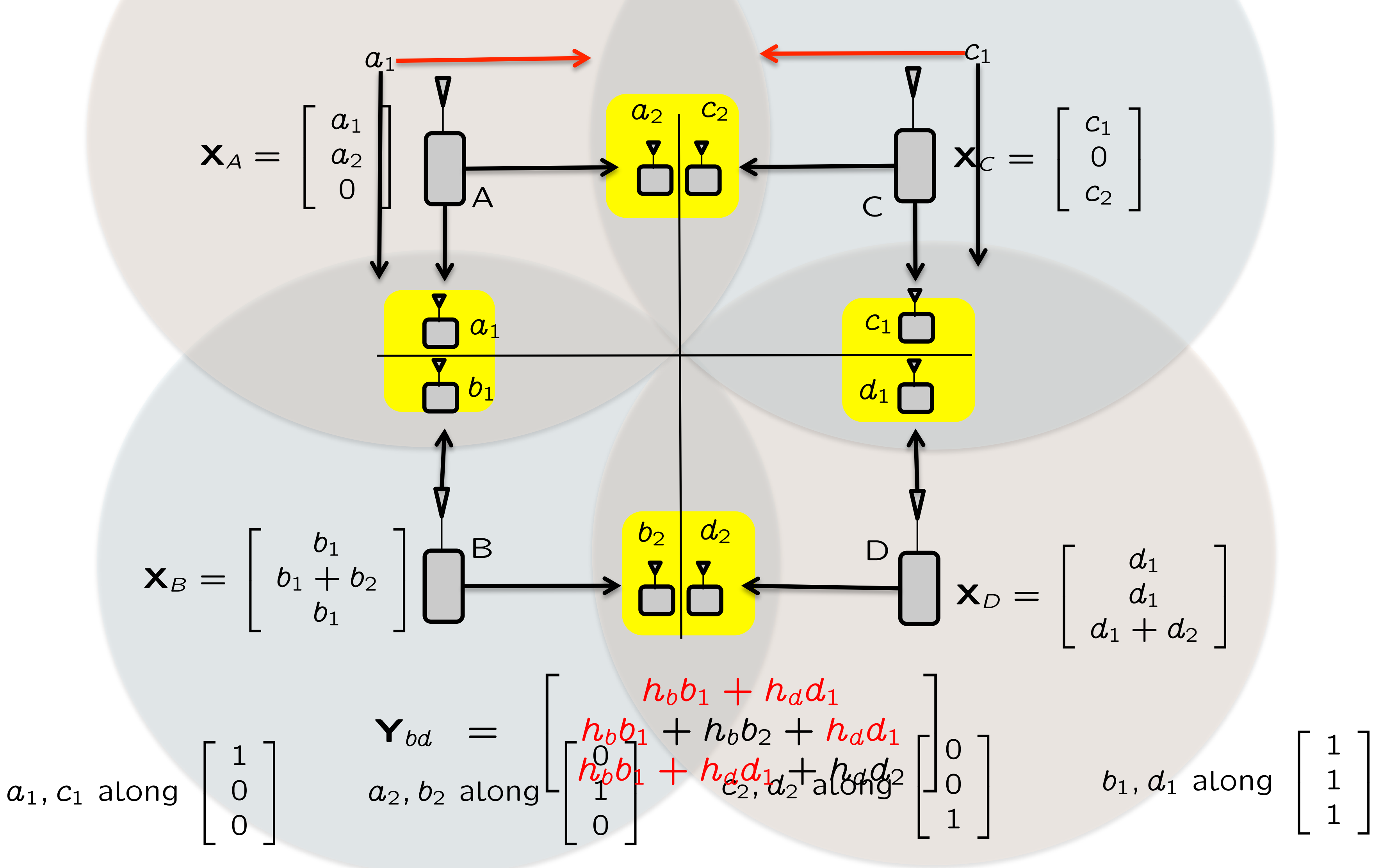
GDoF = ?

With Conventional Schemes: GDoF = 2 (0.5 per cell)

Is this Optimal?

Interference Alignment is the **Key** Even with No CSIT!!

S. Jafar, Topological Interference Management through Index Coding, IEEE Trans. on Information Theory, 60(1):529-568, Jan. 2014.



Without Interference Alignment: GDoF = 2.
With Interference Alignment: GDoF = 8/3.(Optimal)

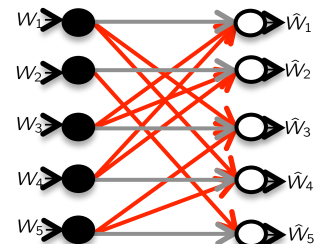
Main Results

TIM is essentially an Index Coding Problem.

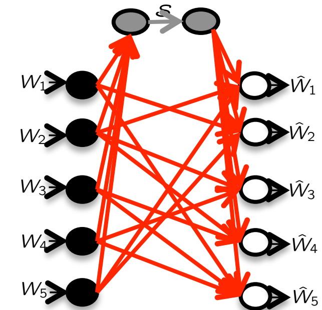
Examples known where IA Gains $\sim O(K^{1/4})$

Many (new) classes solved by IA.

In general an Open Problem.



TIM



Index Coding

Why is TIM / Index Coding a Challenging Problem?

Achievability:

Interference Alignment Perspective [Jafar '13]

Relies on linear schemes

Linear schemes are not always sufficient for index coding

[Blasiak, Kleinberg, Lubetzky '11]

[Maleki, Cadambe, Jafar '12]

Converse:

Need Non-Shannon Information Inequalities

Simplest examples found

[Riis, '13] – (computer search)

[Sun, Jafar, '13] – Interference Alignment Perspective

Practical Principles of Interference Management

Ignore

interference that is sufficiently weak

↓

Treat Interference as Noise

↓

Optimal? TIN

Avoid

interference that is not

↓

TDMA/FDMA/CDMA

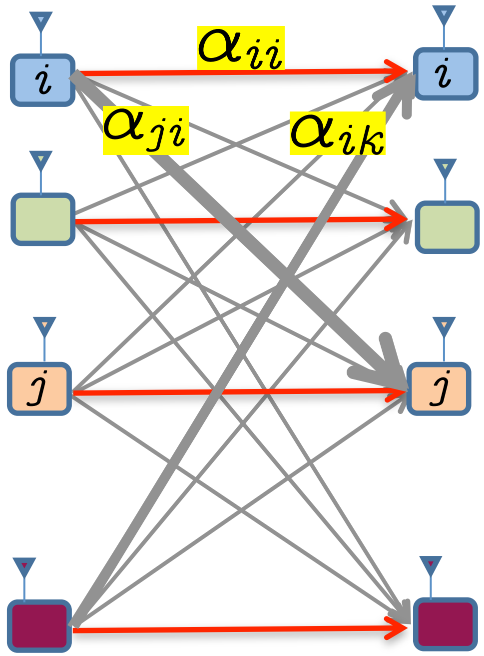
↓

Optimal? TIM

TIN Optimality Condition

[Geng, Naderi, Avestimehr, Jafar, '13]

For the general K-user asymmetric IC, "if for each user the desired signal strength is no less than the sum of the strongest interference from this user and the strongest interference to this user (all values in dB scale), then power control and treating interference as noise is GDoF optimal and achieves the entire capacity region within a constant gap."



K-user Gaussian IC

The Condition

Strongest Interference **from** User i

Strongest Interference **to** User i

$$\alpha_{ii} \geq \max_{j:j\neq i}\{\alpha_{ji}\} + \max_{k:k\neq i}\{\alpha_{ik}\}$$

GDoF Region

(≈ Capacity Region Normalized by $\log(P)$)

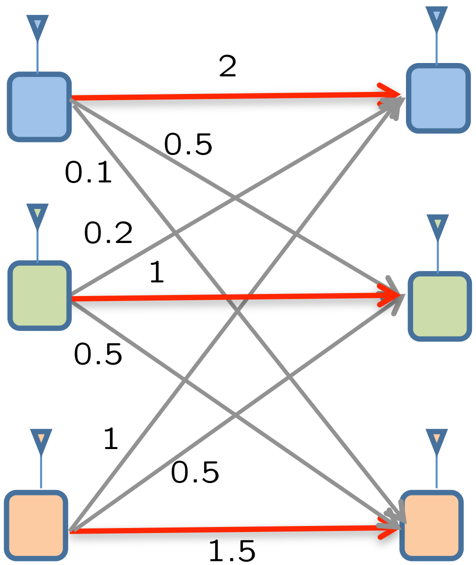
[Geng, Naderi, Avestimehr, Jafar, '13]

The GDoF Region

$$d_i \leq \alpha_{ii}$$

For every cycle:

$$\sum d \leq \sum \alpha_{\text{direct}} - \sum \alpha_{\text{cross}}$$



$$0 \leq d_1 \leq 2$$
$$0 \leq d_2 \leq 1$$
$$0 \leq d_3 \leq 1.5$$
$$d_1 + d_2 \leq 2.3$$
$$d_1 + d_3 \leq 2.4$$
$$d_2 + d_3 \leq 1.5$$
$$d_1 + d_2 + d_3 \leq 3.7$$
$$d_1 + d_2 + d_3 \leq 2.5$$

TIN-condition satisifed: Check.

GDoF Region

(≈ Capacity Region Normalized by $\log(P)$)

The GDoF Region

$$d_i \le \alpha_{ii}$$

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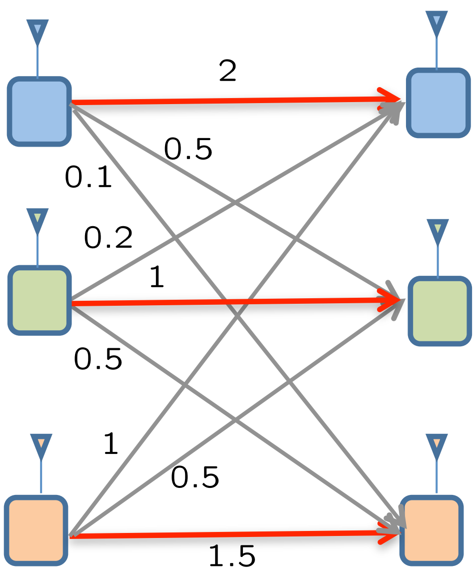

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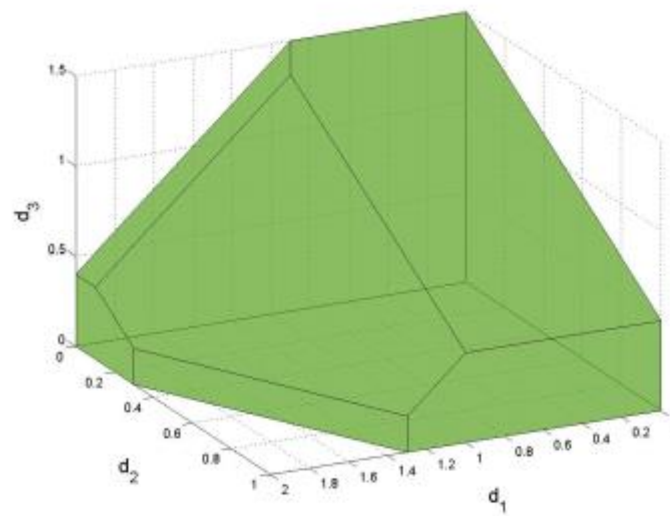
$$d_1 + d_2 + d_3 \le 2.5$$



Observations

Symmetry

[Geng, Naderi, Avestimehr, Jafar, ’13]

The Condition

Strongest Interference from User i | Strongest Interference to User i

$$\alpha_{ii} \geq \max_{j:j\neq i}\{\alpha_{ji}\} + \max_{k:k\neq i}\{\alpha_{ik}\}$$

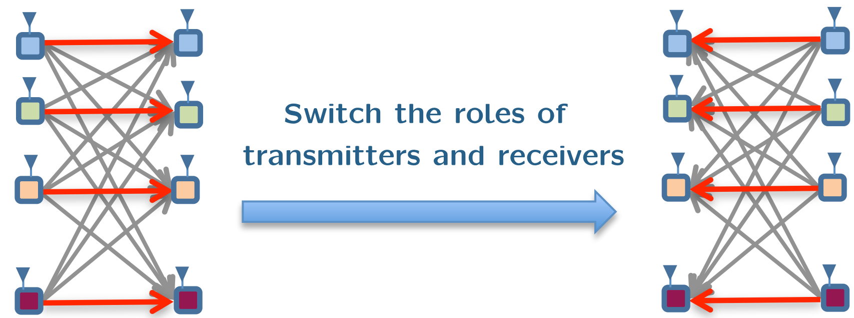
Duality

The GDoF Region

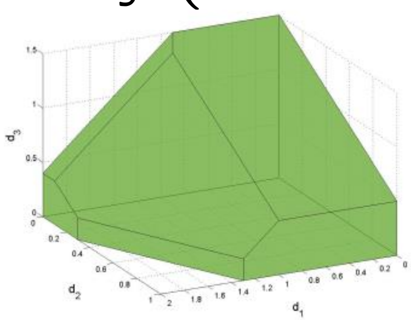
$$d_i \leq \alpha_{ii}$$

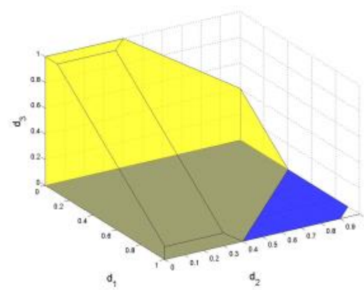
For every cycle:

$$\sum d \leq \sum \alpha_{\text{direct}} - \sum \alpha_{\text{cross}}$$



Convexity (Time-sharing not required)



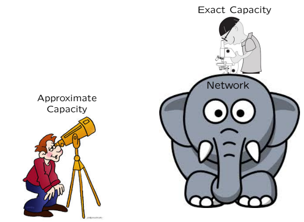


TIN Region – Not Convex in General

Power Optimization Implicit

Conclusion

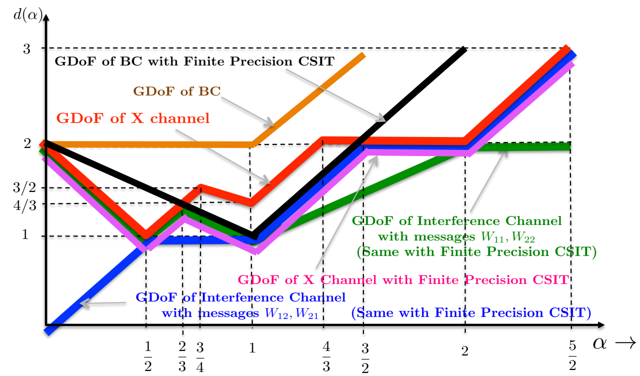
GDoF Under Finite Precision CSIT



2 Users

A. Gholamidavoodi, S. Jafar, Transmitter Cooperation under Finite Precision CSIT: A GDoF Perspective, IEEE Globecom 2015

- Interference Alignment ✘
- Zero-forcing ✘
- Interference Enhancement ✔
- Message-sharing ✔



K Users

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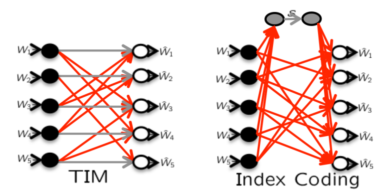
- Interference Enhancement ✘
- Message-sharing ✘
- Interference Alignment ✔
- Index Coding ✔



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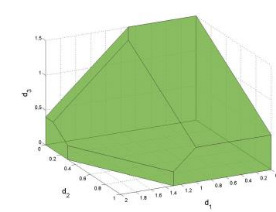
- Alignment due to Saturation ✔
- Treating Interference as Noise ✔



Arbitrary Channel Strengths: Open Problem